RASHTRIYA SANSKRIT SANSTHAN, NEW DELHI

(Kendriya Sanskrit Vidyapeetha, Tirupati Series No. 29)

SIDDHĀNTA ŚIROMAṆI

OF

BHĀSKARĀCĀRYA

English Exposition and Annotation in the light and language of modern Astronomy

By

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KENDRIYA SANSKRIT VIDYAPEETHA

TIRUPATI

1980

First Edition 1980

Price Rs.

Published for the
RASHTRIYA SANSKRIT SANSTHAN
Ministry of Education, S. W. & Culture.,
Government of India,
New Delhi

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TIRUPATI (A.P.)

Printed by the Rathnam Press, Madras-600 001.

राष्ट्रियसंस्कृतसंस्थानम्, नवदेहली

[केन्द्रीयसंस्कृतविद्यापीठग्रन्थमाला, संख्या-29]

भास्कराचार्यविरचितः

सिद्धान्तशिरोमणिः

डा० धूलिपाल० अर्कसोमयाजिकृतया आधुनिकखगोलशास्त्रानुसारिण्या
आङ्गलभाषाव्याख्यया अलंकृतः

संपादकः
डा० धूलिपाल० अर्कसोमयाजी

केन्द्रीयसंस्कृतविद्यापीठम्, तिरुपतिः

1980

प्रथमप्रकाशनम् 1980

मूल्यम्—रू.

शिक्षासमाजकल्याणमन्त्रालयान्तर्गतदेहलीस्थ-राष्ट्रियसंस्कृतसंस्थानस्य कृते
तिरुपतिस्थकेन्द्रीयसंस्कृतविद्यापीठस्य प्राचार्येण
डा. मल्लादि द. बालसुब्रह्मण्यशर्मणा प्रकाशितः
रत्न प्रेस्, मद्रास् इत्येतैः मुद्रितश्च ।

DEDICATION AT THE FEET OF LORD VENKATEŚVARA

समर्पणम्

श्लो. 1. त्वत्पादाब्जदिदृक्षया हिमगिरिप्रस्यन्दिगङ्गाझरी

सङ्काशप्रवहज्जनावळिरहोरात्रं च जोघुष्यते ।

गोविन्देति विमुक्तकण्ठमसकृत् श्री वेङ्कटेशप्रभो !

त्वत्पादाब्जसमर्पिता कृतिरियं भक्त्या प्रसूनायताम् ॥

श्लो. 2. यः सिद्धान्तशिरोमणि र्विजयते श्रीभास्करार्यस्य सः

व्याख्यातो बहुभिस्तथाऽपि मयका व्याख्यायतेऽसौ पुनः ।

व्याख्येयं मणिदीधितिर्विरचिता यादृक्क्रमेण प्रभो !

जानीषे भगवन् ! त्वमेव तमिमां त्वत्पादयो रर्पये ॥

श्लो. 3. पुराचार्यैः प्रोक्तं यदपि च नवीनै स्तदखिलम्

विचार्यैव व्याख्या व्यरचि भगवन् ! भास्करकृतेः ।

यदीयं व्याख्या मे सहृदयबुधाह्लादनकरी

तदाऽहं धन्यः स्यामियमपि कृतिर्मे सफलिति ॥

DEDICATION

English Translation of the Sanskrit verses

1. Oh ! Lord Venkateśvara ! I offer thee this work of mine as a flower at Thy feet, who art being approached day and night by throngs of people in a flow like that of the Ganga, jumping down the heights of the Himalayas, crying out full-throated one of Thy names 'Govinda' in shrill imploring voices, surging out from the depths of agonizing hearts !

2. The great work named Siddhānta Śiromaṇi of Bhāskarācārya, was indeed commented upon by a host of scholars both ancient and modern. Yet, I not a great scholar make bold to comment once again. Oh ! Lord Venkateśvara ! *Thou alone Knowest* and no mortal does, what stress and strain I underwent while working on this commentary. Hence I dedicate this at Thy feet alone not to any mortal, however great he may be !

3. This commentary has been written by me carefully understanding the depths of both the ancient and modern systems of astronomy. If this could invoke the pleasure of well –meaning scholars, then I deem myself fortunate, and that my toil will have been well rewarded.

श्लो. 4 यो ग्रन्थो महता श्रमेण लिखितः प्रवैर्विशेषैर्युतः

यत्सारोऽपि विपश्चितां सुमनसां सूक्ष्मेक्षिकागोचरः ।

यः कश्चिद्यदि पिष्टपेषणमिदं स्यादित्यबुध्द्वा वदेत्

यद्वा किञ्चिदवेक्ष्य सोऽत्र निहितान् भावान् कथं ज्ञास्यति ॥

श्लो. 5. यः सिद्धान्तशिरोमणिं न पठितः श्रीभास्करार्यस्य य

श्चाऽऽस्मानं लघुपुस्तकेक्षणपरः चेत्पण्डितंमन्यते ।

रङ्गत्तुङ्गतरङ्गितं हिमगिरिप्रस्यन्दि गङ्गाजलम्

हित्वा तेन सुपङ्किलं किल जलं स्नातुं समाश्रीयते ॥

व्याख्याता

डा. धूळिपाळोह्वा अर्कसोमयाजी

4. This work indeed sucked the very blood out of me. It contains many a new detail unnoticed hitherto. Only unbiased scholars could understand the essence of this work. If somebody pronounces unknowingly that this is all a repetition of what has been already written, simply having turned out a few pages, how could he know where I showed new aspects of the genius of Bhāskara?

5. He, who does not study the Siddhānta Śiromaṇi of Bhāskarācārya, and feels that he is a scholar reading a few other sub-standard books on Hindu astronomy, does verily go to bathe in a dirty pond, ignoring the holy Ganga, jumping down the heights of the Himalayas in surging and dancing billows!

D. ARKASOMAYAJI

व्याख्यात्रभिजनः

श्लो. 1. आन्ध्रेषु प्रथितं विराजति पुरं गोदावरीतीरगम्
श्रीमद्राजमहेन्द्रमित्यभिहितं यस्यान्तिके भ्राजते ।
ग्रामः श्रीवलिचेरु नाम विबुधैर्देदीप्यमानस्सदा
देवब्राह्मणयायजूकविलसद्वंशे जनिस्तत्र मे ॥

श्लो. 2. माता मे मङ्गमाम्बा सततमपि पतिं सेवमाना च दुर्गाम्
देवीं नित्यं भजन्ती निजतनुमनयत्सेव्यसेवां चरन्ती ।
विद्वान् बापार्यनामा श्रुतिविहितपथे सञ्चरन् मे पिताऽऽसीत्
मातापित्रो स्तयोर्मे पदभजनमहो बाल्य एव व्यरंसीत् ॥

3. श्रीमद्वेङ्कट रामाख्यमग्रजं कर्मठं स्तुवे ।
प्राविक्षं वेदवेदाङ्गवाङ्मयं यत्पदान्तिके ॥

4. द्वितीय मग्रजं वन्दे सुब्रह्मण्याह्वयं ततः ।
देवब्राह्मणपूजायां जीवितं यस्य यापितम् ॥

5. वेदवेदाङ्गविद्यायां बाल्ये हन्त ! प्रवेशितः ।
जीविकायै ततो हूणभाषामध्यापितोऽभवम् ॥

BIOGRAPHICAL

1. In the state known as the Andhra Pradesh, there is a famous city called Rajahmundry on the banks of the holy Godāvarī. Not far away from this City, there is a village known Velicheru, teeming with Vedic and Sanskrit scholars. There I had my birth in a family reputed for a religious conduct.

2. My mother, by name, Mangamāmbā led her life both in the worship of my father, and her favourite Deity Kanaka Durgā. My father, a Vedic scholar conducted his life on a rigorous Vedic path. Alas! It was not given to me to serve them for long!

3. I bow to my eldest brother Śri Venkata Rāma who leads a rigorously religious life, and who it was, that initiated me into a study of the Vedic and Sanskrit lore.

4. I bow to my next elder brother by name Subrahmaṇya Somayāji who has passed away having spent his life in worshipping God and godly Brahmins.

5. Alas! Though I was initiated into the Vedic and Sanskrit lore in my boyhood, I was later distracted into the secular English education only to eke out my livelihood, as if this is the Summum Bonum of Life.

6. मादृशाः कति वा विप्रकुलोद्भूताः श्रुतिस्मृतीः ।
हित्वा हन्ताङ्ग्लविद्यायां प्रविष्टाः कलिकालतः ॥

7. श्रीनिवासमहं सेवे योऽत्र पर्वतमूर्धनि ।
येन स्वपादसेवाया आनीतः स्वपदान्तिकम् ॥

8. यत्सेवा च दिवारात्रं धर्मपत्न्या कृता मम ।
आवामत्र निवासार्थमानयत् स्वगृहादिव ॥

9. स्वस्ति तस्मै भवेद्राज्ञे रक्षितुं यो व्यवस्यति ।
पारावारगभीरं तद्वेदवेदाङ्गवाङ्मयम् ॥

व्याख्याता
धूळिपाळोह्वा अर्कसोमयाजी

6. How many like me, born in Brahmin families, are being proselytized into the English system of education burying in the Bay of Bengal all that is grand in the Vedic and Sanskrit lore.

7. I raise my hands in supplication to Lord Venkateśvara, who took His abode on the Seven Hills as though to look down upon this mundane and irreligious world. It was He that has brought me here, to worship Him residing at His very feet.

8. I feel it was the deep devotion of my wife to Lord Venkateśvara that has brought us here, dislodging us from our home and hearth.

9. May those administrators flourish, who exert and strive to save the ocean—like Vedic lore from the yawning mouth of Oblivion!

D. ARKASOMAYAJI

FOREWORD

Bhāskarācārya, the Second (c. 1100 A.D.) was one of the greatest mathematicians and astronomers of the World. He anticipated modern theory on the convention of signs (minus × minus = plus) and KEPLER'S method of determining the surface and the volume of a sphere. Six hundred years before the calculus of LEIBNITZ and NEWTON, Bhāskara worked at the differential coefficient. He raised and solved the problem —

$$67\,x^2 + 1 = y^2$$

—a problem which FERMAT resolved after 500 years. The credit goes to Bhāskarācārya for having delineated the image of *Jyotiṣa* in its proper contours and graces.

The *Siddhāntaśiromaṇi* is often said to be the *magnum opus* of Bhāskarācārya. This monumental treatise consists of four parts: (1) *Līlāvatī* (arithmetic), (2) *bījagaṇita-* (algebra), (3) *Golādhyāya-* (Trigonometry including spherical trigonometry) and (4) *Grahagaṇita-* (Planetary motion).

This work was edited with the *Vāsanābhāṣya-* of the author by Bapu Deva SASTRI. Muralidhar JHA brought out two commentaries—the *Vāsanāvārttika-* of Nṛsiṁha (1621 A.D.) and the *Marīci* of Munīśvara (1635 A.D.) on the first Chapter of the *Gaṇitādhyāya* (1917). Girija Prasad DVIVEDI's commentaries in Sanskrit and Hindi (volumes I & II) appeared in 1911 and 1926.

Bapu Deva SASTRI and WILKINSON published an English translation of the text in 1861. Yet the mathematical aspect of the *Siddhāntaśiromaṇi* has remained a *terra incognita* to students of Hindu Astronomy. While *Phalita-Jyotiṣa*-continues to be studied in traditional Sanskrit institutions, the *Gaṇita*-side of Astronomy is reduced to a secondary position.

Realising that specialists in the twin fields of Mathematics and Astronomy have been diminishing day by day, Kendriya Sanskrit Vidyapeetha at Tirupati has started a project entitled, "Coordination of Sanskrit and Ancient Indian Sciences". Under this scheme, Dr. Arka Somayaji has now come forward to give an exposition and annotation of the *Siddhāntaśiromaṇi* in simple English based on the language of modern Astronomy for the benefit of the students and scholars interested in the *Khagola-śāstram.*

Dr. Arka Somayaji is an eminent scholar in Mathematics and Dynamic or Spherical Astronomy. He hails from a family of "Siddhāntin-s". To whet his appetite in learning the mnemonical methodology for compiling an almanac, he studied Mathematics and Jyotiṣa (including Spherical Astronomy). He learnt the *Taittirīya-saṁhitā* under his brother, Dhulipala Venkatarama Avadhani of Rajahmundry. He wrote a thesis on "*A Critical Study of the Ancient Hindu Astronomy*", which was published in 1972. He has to his credit eight works including the *Jyotirvijñānam* (1964) and two Sanskrit poetical compositions (*Brahmāñjali* and the *Hanumat-vijaya-*). He won the President's award in 1974.

In order that critical editions of rare and valuable texts on *Jyotiṣa* be brought out, the Tirupati Vidyapeetha appointed Dr. Arka Somayaji as Reader in Hindu Astronomy. Accordingly the Vidyapeetha has undertaken the publication of his English annotation of the *Siddhāntaśiromaṇi.* I trust this treatise will go a long way not only to project India's image in the World of Mathematics and Astronomy, but also inspire scholars from the transoceanic distance to listen to the jungle roar of the ancient Indian Wisdom.

KENDRIYA SANSKRIT VIDYAPEETHA
TIRUPATI
October 21, 1980

M. D. BALASUBRAHMANYAM
Principal

PREFACE

From the Mathematician's point of view, Bhāskarācārya's Siddhānta Śiromaṇi contains all that was beautiful in the Ancient Hindu Astronomy. I am aware that a translation of this work into English was done long ago by M. M. Bapudeva Śastry and Wilkinson; but I did not have the good fortune of having a copy in my hands all these years. I did go through the translation once long ago, but it gave me the impression that all the beauties there that appeal to a Mathematician were not brought out fully. There are, however, a good number of Sanskrit commentaries both ancient and modern but even they, in my humble opinion, have not done full justice to the elucidation of Bhāskara's mathematical genius. Further misinterpretations are not infrequent in some of those books, as will be pointed out in the course of this book.

What has sponsored me to undertake to write an English commentary, (I may add that a Sanskrit commentary also has been written by me on this work and has been awaiting printing) is essentially that innumerable modern professors of Mathematics complain many a time that there is no such a presentation of Siddhānta Śiromaṇi in English as will enable them to assess the Mathematical content of it in the light and language of modern Astronomy. Hence, I have sought to produce a fresh commentary, which I hope will meet the desire of such professors and students of Mathematics. I may add here that this work of mine seeks to present only the *mathematical side* of Siddhānta Śiromaṇi. It is no history of Hindu Astronomy, where the originality of the Ancient Hindu Astronomers is sought to be evaluated. I may confess that I am no historian.

In the course of this book, I shall have occasion to quote from most of the Astronomers of ancient India like Aryabhaṭa I, Varāhamihira, Lalla, Aryabhaṭa II, Brahmagupta, Bhāskara I, Munjāla, Vateśvara and Śrīpati not to speak of some others, whose books are available in print or manuscript.

One thought that lurks in my mind, I make bold to present in this preface. I say 'I make bold' because, there are some historians of Hindu Astronomy, who are too ready to attack me if I claim that there must have been not a primitive astronomical activity in ancient India prior to Aryabhaṭa I. The reason for their attack is that prior to Aryabhaṭa's work, only one crude Vedānga Jyotiṣa has come to light. Also some of these historians have a strong impression that the galaxy of Hindu Astronomers ranging from Aryabhaṭa derived an incentive from a foreign source especially the Greek. If such historians agree to keep an open mind, I make bold to present my thought as follows. In my humble opinion there must have been considerable astronomical activity in ancient India even prior to Aryabhaṭa. This is borne out by the following expressions of Aryabhaṭa and others. Aryabhaṭa says[1] that he dived into the then extant astronomical lore, which got mixed up with mathematical and non-mathematical (mythological or otherwise) knowledge, and by his intellect and the grace of his Goddess, brought-out the truly mathematical. Varāhamihira[2] says that he was codifying the then extant five Siddhāntas, out of

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1. सदसत्ज्ञानसमुद्रात् समुद्धृतं देवताप्रसादेन
सद्ज्ञानोत्तमरत्नं मया निमग्नं स्वमतिनावा — Verse 49—Goḷapāda

2. पूर्वाचार्यमतेभ्यो यद्यत् श्रेष्ठं लघुस्फुटं बीजम्
तत्तदिहाऽविकलमहं रहस्यमभ्युद्यतो वक्तुम् — Verse 2—Ch. I.

पौलिशकृतः स्फुटोऽसौ तस्यासन्नस्तु रोमकप्रोक्तः
स्पष्टतरः सावित्रः परिशेषौ दूरविभ्रष्टौ — Verse 4—Ch. I.

which the 'Sāvitra' was more accurate. Brahmagupta[3] says, again, that he was giving a clear presentation of the ancient Brahma Siddhānta which got obsolete by a long lapse of time. In the wake of these statements and a number of others, is it not right to construe that there did exist some ancient astronomical texts which are lost to us. If, as asserted by a host of modern interpreters, we think that there was only the crude Vedāngajyotiṣa, before Aryabhaṭa, does it not tantamount to saying that these three great astronomers (leave others) were impostors, who, having derived their knowledge from a foreign source, simply claimed that there were existent before them, Saura, Brāhma and some other Siddhāntas which dealt with Graha-gaṇita. It is uncharitable to say that three rational astronomers were sueh impostors. So, we must conclude that there did exist some astronomical activity, which we have no right to call primitive like the Vedāngajyotiṣa. Some might think that there might be existing some other texts in between the times of Vedāngajyotiṣa and Aryabha but that at the time of the Vedāngajyotiṣa[4] (roughly 1180 B.C.) astronomy in India was that crude. Even this conclusion need not be

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"I am going to tell clearly what has been, the best, what has been kept secret and briefly-worded in a nut shell from out of the works of ancient Acāryas".

"The Siddhānta of pauliśa is allright, the Romaka is very nearly the same, but the Sūryasiddhānta is more accurate, whereas the two others Vāsiṣtha and paitāmaha are crude".

3. ब्रह्मोक्तं ग्रहगणितं महता कालेन यत् खिलीभूतम्
अभिधीयते स्फुटं तत् जिष्णुसुतब्रह्मगुप्तेन ॥ Verse 2—Ch. I.

4. As per statement आश्लेषार्धात् दक्षिणमुत्तरमयनं रवेर्धनिष्ठाद्यम् of Varāhamihira quoting the meaning of the verse.
प्रपद्येते श्रविष्ठादौ सूर्याचन्द्रमसावुदक्

सार्पार्धे दक्षिणा......Verse 7—Vedāngajyotiṣa, the vernal equinoctial point was at the beginning of Dhaniṣtha, which means that the time was 1180 B.C. approximately.

correct, for the simple reason that, even today, crude works exist side by side with advanced works. The simple fact that a Vedāngajyantiṣa belonging to 1180 B.C. has been unearthed and nothing else, is not a complete proof that there did not exist more advanced texts some where else in such a big country like India especially when locomotion or transport was difficult.

Just one more thought, I place before the learned historians, which is pertinent to this context. According to geology, biology and some other similar modern sciences, the first man came into the cosmic picture some millions of years ago. If that be so, how is it that we say that man remained stupid all these years and happened suddenly to blossom into a genius one fine morning round about Eighteen fifties (1850 A.D.) whereafter came all Scientific discoveries in a series, as if by the waving of a magic wand? It could not be so, as if, we are the chosen few of God. Civilizations must have been there, which got buried in the bosom of the earth, as has been revealed by the Mohenzadaro excavations. Hence we are not right in saying that the ancient Hindu civilization was so primitive at 1150 B.C. as is reflected in the Vedānga Jyotiṣa. 'This is another and more important reason that this author makes bold to place before the historians as to why he (the author) does have faith in the statements of Aryabhaṭa, Varāhamihira and Brahmagupta who said that they were peacing out the knowledge contained in the then extant works signifying at the same time that many more books were lost even to them.

In this commentary of mine, I have chosen to keep the commentary away from the Sanskrit text and Bhās-kara's own Vāsanā Bhāṣya, for, otherwise the book grows bulkily unwieldy. I have chosen to keep silent over passages which do not call for a mathematical elucidation. Here and there I have chosen to present what modern astronomy presents in some particular contexts, so that, mathematics students who do not happen to study modern astronomy may have a better perspective of the ancient Hindu astronomy, presented in Juxtaposition to the corresponding modern treatment. However, I do not propose to enter into the intricacies of modern astronomy which are not called for to elucidate the text.

Before concluding, it is my sacred duty to thank the Raṣtrīya Sanskrit Samsthan, under the Ministry of Education, Social Welfare and Culture, Government of India, for having accepted my work under the publication series of K. S. Vidyapeetha, Tirupati and the late-lamented Dr. M. Ananthasayanam Ayyangar, former Chairman of KSV and Ex-Governor of Bihar, who encouraged me in my studies on Hindu Astronomy. Dr. M. D. Balasubrahmanyam deserves my thanks for the encouraging interest he has shown in the publication of this annotation,

D. ARKASOMAYAJI

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ओं श्रीमहागणपतये नमः, श्रीगुरुभ्यो नमः

सिद्धान्तशिरोमणौ गणिताध्याये मध्याधिकारे कालमानाध्यायः

Verse 1. May the Sun at once give expression to our tongue in meaning ful words-that Sun, who rises to protect this world, whose duty it is to expel darkness, who is reported to be the Spouse of the lotus-creeper, who purges out the sins of all those that supplicate him, and on whose rising take place Vedic sacrifices and thereby the gods in heaven headed by Indra get feasted.

Commentary (*Comm.*). Needs no Commentary.

Verse 2. Excels that blessed Brahmagupta, the son of Jishnu, who is hailed as the crest-jewel of Mathematicians; excel also those like Varāhamihira who were the authors of well known works, who were adepts in reasoning, and usage of beautiful expression and on a study of whose works, one like me even of lesser intellect, will be able to produce monumental works.

Comm. Needless to comment. However, we may state one thing. Bhaskara's quoting the name of Brahmagupta at the very outset, and that too with reverence, informs us that he is going to accept the Āgama of Brahmagupta in preference to that of others like Aryabhata. This means that he is going to adopt the number of revolutions of planets in a Yuga and such other astronomical constants as were adopted by Brahmagupta. This point will be clarified later.

Verse 3 This blessed author Bhaskara is now writing the work named Siddhānta-Siromani, the crest-jewel of all astronomical works, for the pleasure of good minded astronomers, after having bowed to the lotus-feet of his father, from whom he has derived his knowledge; this work, containing good metres, will be easy to understand, besides being flawless and clear, and will enable intelligent readers to develop their ability to understand things.

Comm. Not neeessary.

Verse 4. Ancient astronomers did write, of course, works abounding in intelligent expression; nonetheless, this work is started to give expression to some lacunae in their works. I am going to make amends for the deficiencies of the older works and these improvements will be found here and there in their respective places; So, I beseech the good-minded mathematicians to go through this entire work of mine also (for, otherwise, they may not locate my own contribution).

Comm. Not necessary.

Verse 5. May the good people be pleased with the particulars of my contribution! May the ill-minded people also derive pleasure out of ridiculing me with ignorance unable to understand my contribution!

Comm. Not necessary.

Verse 6. A Siddhānta work ie an astronomical treatise is such a one which deals with the various measures of time ranging from a Tṛti (to be explained shortly) upto the duration of a Kalpa which culminates in a deluge; planetary theory, arithmatical computations as well as algebraical processes, Questions with respect to intricate ideas and their answers, location of the earth, the stars and the planets, and description and usage of instruments.

Comm. Not necessary.

Verse 7. Though one knows astrology and that part of the science of Jyotisha which is known as Samhita (and which deals with various subjects like *Muhurtas* i.e. auspicious moments to be prescribed for various functions, Desarishtas ie Calamitous occurences to the countries etc.) which form a part of the Science, he cannot answer so many intricate problems pertaining to Artronomy. Such a person, who does not know the astronomical part of the science, which abounds in innunerable reasonings, is one like a king depicted in a drawing, or a lion fast tied to a pole.

Comm. Not necessary.

Verse 8. The Science of Jyautisha without Astronomy, is like a king's army without roaring elephants though excelling in horses etc; is like a garden without mango trees, or like a lake without water, or again like a lady parted with her newly married lover.

Comm. Not necessary.

Verse 9. The Vedic lore prescribes Sacrifices to be performed; these sacrifices are based upon a knowledge of appropriate time to perform them. This science of astronomy gives a knowledge of time; hence it has been reckoned as one of the six Vedāngas or limbs of the Veda.

Comm. Not necessary.

Verse 10. (Out of the six Vedangas) The science of grammar is like the face of the person of the Veda, the science of Jyautisha takes the place of the eyes, the Nirukta that of the ears; the Kalpa that of the hands; the Siksha that of the nose and the Chandas the place of the feet.

Comm. Not necessary.

Verse 11. This science of Jyautisha being depicted as the very eyes of the Person of the Veda, so it has been acclaimed as the most important of the six Angas or limbs of the Veda, in as much as, even if a person be endowed with limbs like the ears, nose etc, if he be devoid of vision, he could not do anything.

Comm. Not necessary.

Verse 12. (Since this Science of Astronomy has been declared as the most important of the Vedangas) hence this has to be studied by the Dwijas (ie Brahmins Kshatriyas and the Vaishyas who form the three higher castes), also because it is sacred, secret and the best discipline. By so doing they would acquire Dharma, Artha, Kāma as well as fame (Life is depicted as having a four-fold purpose out of which, Dharma, Artha and Kāma form the first trio, Moksha being the ultimate goal of life).

Comm. Not necessary.

Verses 13 and 14. The creator having created the stellar circle along with the planets, placed the latter at the beginning of the circle, put them in constant revolution, at the same time putting the extreme two stars (on either side) in a fixed position.

Comm. Bhaskara's own commentary Vāsanā Bhasha under this verse mentions the following points, which are to be noted. The twenty seven stars known as Aswini, Bharani etc. occupy positions roughly at equal distances along the Zodiac, arranged from west to east. The planets were all placed in the beginning of the stellar circle in such a way that they were in a straight line the moon, Mercury, Venus, Sun, Mars, Jupiter and Saturn occupying consecutive positions from the earth in increasing distances, not of uniform measure. The circle of stars known as the Zodiac lies far behind all the planets. There is a wind known as pravaha which keeps the entire Zodiac, as well as the planets below going round and round in the Westerly direction. At the same time, the planets while participating in this westerly motion, have themselves individual motion towards the east. Two stars, are placed one at the north-pole and the other at the south, and they are fixed[1]. The diurnal motion due to pravaha is far greater than the individual motion of the planets among the stars in a direction from west to east ie the direction opposite to that of diurnal motion.

In this context, it is worth-noting that Aryabhata I mentioned the diurnal rotation of the earth in the verse "अनुलोमगतिः..." Bhaskara must have been aware of this; yet, in spite of his rational outlook in all matters, misguided himself in this respect (vide verse 3 Madhyagativāsanā-Goladhyaya), apparently for fear of tradition.

Verse 15. The first Mahayuga, the first year, the first day of the bright half of the first month named Madhu,[2] all of them began simultaneously at the Sun-rise[3] at Lankā on Sun-day, at the beginning of the first Kalpa which marked the beginning of creation.

Comm. Though, in the Commentary before, Bhaskara gave expression to the fact that Time is eternal with no beginning and no end, herein he mentions the point of Time when creation commenced. So, at the back of his mind, the concept of Time arose only at the beginning of creation, whereas before creation, as well as after deluge the आत्यन्तिकप्रलय, there was and would be neither the concept of Time nor space. In other words, both space and Time are manifest only after creation, and get extinguished after deluge.

Verses 16, 17 and 18. The unit of Time named Tatpara is 1/30th of what is known as Nimeṣa or the time taken during the fall of an eyelid; One-hundredth of a Tatpara is known as a Truṭi ie the time taken to pierce a lotus-leaf with the finest needle. Eighteen Nimeṣas are equal to a Kāṣṭhā. Thirty Kāṣṭhas are equal to a Kalā, thirty Kalās are equal to one sidereal ghaṭī. Two ghatīs make a Kshana and thirty Kshanas make a sidereal day. Thirty sidereal days are equal to a sidereal month and twelve sidereal months make a sidereal year (not the sidereal solar year). The Zodiac, divided into twelve rāśīs, and 360 degrees, a degree divided into 60 minutes of arc and a minute divided into 60 seconds of arc all correspond to the year and its successive divisions.

Comm. In the Commentary under these verses, Bhaskara gives further details of division of time as follows. The time taken to pronounce a guru, i.e. double the time of pronouncing a short vowel, is one-tenth of a Prāna, which is the time required by a healthy person to inhale and exhale once. Six Prānas make one Vighatī and sixty ghatis make one sidereal day. It may be clearly noted here that this sidereal month consisting of thirty sidereal days is not the sidereal month that is the time taken by the Moon to go round the Zodiac, nor the sidereal year defined above is the time that the Sun takes to go round the Zodiac once in his apparent annual motion. To distinguish these latter divisions of time, we shall use the nomenclature sidereal lunar month and sidereal solar year. There are further other divisions of time which will be later elucidated.

Verses 19, 20. The time taken by the Sun to complete one revolution with respect to the stars goes by the name 'The sidereal solar year'. This will be a day for the gods and demons. The time that elapses between two consecutive new moons or conjunctions of the Moon with the Sun is called a Chāndra-māsa or a lunar month or simply a lunation. This again is the day of the Pitṛs or the Manes.

The time that elapses between two consecutive Sun rises at a place is termed the Sāvana day or civil day. This is called Saura-Sāvana day and it is also the day of the earth.

The sidereal day is the time taken by the stars to go round the earth once. It is called Nākshatra-dina.

Comm. In Hindu mythology gods are supposed to reside at the north-pole, where one civil day is the same as one sidereal solar year for the other places; also the demons are supposed to reside at the south pole so that their civil day also is equal to a sidereal solar year. But, what is day to the gods is night to the demons and vice-versa.

If we call the earth, the moon of the moon so to say, which it is so, when it is the moment of new moon for the earth, it is the moment of Full Moon to the Moon. Thus what is a Chāndra-māsa to the earth may very well be called with respect to the Moon a Bhauma-māsa. In Hindu mythology the manes are supposed to take residence on the surface of the Moon. We know from modern astronomy that the moon revolves about her axis once roughly in a lunation. Thus for 'the man on the moon', a day is roughly equal to our lunation. So if the manes were to reside on the moon, their day is equal roughly to a lunation of ours. We say '*roughly*' because as we see under the chapter of the lunar eclipse, the moon *almost* shows the same face to the earth on account of what are called 'librations in longitude'. On this count the time of rotation of the moon does not exactly coincide with the time of a lunation.

A civil day for a place is also the civil day for every place of the earth so that it is called the earth's day. By 'day' here we do not mean the time when the Sun is above the horizon, for that time differs from place to place on the earth. A civil day is the sum-total of the duration of day and the duration of night for any place and it will be seen that this is the same for the entire earth except at the places having perpetual day. The word Saura-Sāvana is used to signify that the time that elapses between two consecutive rises of any other planet is termed the Sāvana day pertaining to that planet. Thus the Chāndra Sāvana day is roughly equal to 24 hrs–52[1] and this is the longest of the Sāvana days pertaining to the planets. The shortest Sāvana days is that of the Saturn, since Saturn moves a very little along the ecliptic and his Sāvana day is therefore just a little longer than the sidereal day. This nomenclature brings in the phenomenon that the Sāvana day of a retrograde planet happens to be less than a sidereal day. But in a given time, which is sufficiently long like a yuga, the Sāvana days of a planet are equal to the number of sidereal revolutions in that period minus the number of the planetary revolutions in the same period. This will be constant for a given planet in such a long period, though individual days happen to be some shorter and some longer than a sidereal day.

The modern sidereal day is the time between two consecutive rises of the equinoctial points and as the equinoctial points have a slow retrograde motion, the modern sidereal day is just a little shorter than the Hindu sidereal day, which does not take cognizance of the revolution of the equinoctial points round the earth in reckoning diurnal motion. The Hindu astronomers speak of the revolution of the stars only around the earth in the context of diurnal motion. It will be noted that the Saura-Savana day or the civil day will not be of the same duration, since the Sun has unequal motion amongst the stars from day to day. When the Sun is in perigee and has the max. daily velocity, his Sāvana day at that moment is the longest and the Saura-Sāvana day when the Sun is in apogee will be the shortest.

Though in Indian Chronology a day is divided into sixty ghatis for convenience, these ghatis are evidently longer than the Nākshatra-ghatis or the sidereal ghatis, which are of a fixed duration. Thus a Sāvana ghati is a little longer than a Nākshatra-ghati and what is more, a Saura-Sāvana ghati is of a variable duration from day to day, the variation being of course very small.

The concept of a month originally arose out of the phenomenon of new-moons, for, this phenomenon alone appeals to every lay man, when he could not sight the Moon. The concept of an year arose originally out of what is called a tropical year, which is the time between two consecutive conjunctions of the Sun with an equinoctial point and which it is that makes the seasons recur. Thus the primitive man must have had the concept of an year when he once saw the mango trees blossoming and again when he saw them blossom. This is why the Hindus celebrate the new year's day with eating the neem flower with unripe mangos, which goes by the phrase Nimba-Kusuma bhakshaṇam or eating the neem flower. In the Vedic times, however, the year began with the luni-Solar month called Mārga-Sīrṣa, which brings in the new year crops. In the Vedic sacrifices, there is thus what is called the Āgrahāyaṇeṣti, where the word Āgra-hāyaṇa means the Mārga-Sirṣa month. The etymology of the word is that अग्रे हायनं यस्य तत् आग्रहायणम् ie the year is ahead of this month, which therefore is the beginning month of the year. This is also why the ancient lexicon named Amarakos'a enumerates the months from Mārga-Sīrṣa. This is the month when the full moon occurs when the Moon is in the star Mṛgas'ira. We have also an inkling from this that the vernal equinoctial point was probably situated in the star Mṛgas'ira. The Veda however enumerates the stars from the star Kritticā, and we have a statement in the S'ata-patha-Brāhmaṇa that "एता ह वै कृत्तिकाः प्राच्यै दिशो न श्च्यवन्ते ie Behold! these stars which go by the name the Kritticas do not deflect from the east-point. As this group of Krittikas is situated on the ecliptic, the statement that they were rising in the east signifies that the Vernal equinoctial point was situated in the Kritticas". Arguing about the situation of the vernal equinoctial point in the so-called Vedic times, the late Lokamānya Bāla Gangā-dhara Tilak concluded that Vedic literature must have had its beginning about eight thousand years ago.

From the original concepts of the month and the year, further concepts of the different kinds of month and the year arose with the advance of astronomical knowledge. We shall deal with these different kinds of the month and the year in their respective contexts.

In these two verses, we have the definitions of Saura-māna, Daiva-māna, chāndra-māna, Paitra-māna, Sāvana-māna and Nākṣatra-māna, six of the nine mānas, ie measures of time.

Verses 21, 22, 23, 24, 25. The four yuga-pādas named Krita, Tretā, Dwāpara and Kali consist of 4 × 432000, 3 × 432000, 2 × 432000 and 432000 mean solar years respectively, the sum total of which consisting of (4+3+2+1) × 43200 = 43,20000 mean solar years, is calleda yuga. Each of the yuga-pādas above are inclusive of what are called their respective Sandhyās and Sandhyāmsās which constitute one-twelfth of their own durations.

A Manu's duration consists of 71 yugas and 14 Manus' duration is reported to be the day-time of Brahma, whose night is also of an equal duration.

The duration of a Manu, known as a Manvantara has a Sandhyā-Kāla on either side, ie before and after, equal to one Krita. If these are taken into account, the day-time of Brahma amounts to one thousand yugas and it goes by the name a Kalpa so that a complete day of Brahma equals two Kalpas. The life-duration of Brahma consists of one hundred years on this scale (where one year = 360 days). This life-duration of Brahma goes by the name Mahā-Kalpa, as reported by elders. In as much as Time was without a beginning and will have no end either, I do not know how many Brahmas have gone before.

Comm. In these verses we are given what is known as Brāhma-māna, the seventh of the nine mānas. Incidentally we are also given the measures yuga-pādas, Manvantaras and a Kalpa. Since in many ancient astronomical works, the revolutions of the planets and the planetary points like nodes, apogees or aphelia are given as integers during the course of a yuga, the concept of a yuga must have arisen as follows. The durations of the sidereal revolutions of the planets and the apogee of the Moon and its node having been ascertained by observation, a period was calculated in which are contained integral multiples of those durations. In other words a yuga of 4320000 mean solar years is construed as the period in which the planets the node and apogee of the Moon make an integral number of revolutions with respect to the stars.

We have excluded here the aphelia and the nodes of the planets, as we shall see later that their sidereal revolutions were not based on observation but by an assumption that those points also must be having an integral number of revolutions during the course of a Kalpa, having started at the beginning of the Hindu Zodiac ie the beginning point of Aswini at the beginning of the Kalpa. On this assumption cited, and using indeterminate analysis the number of their sidereal revolutions were got as reported by Bhāskarāchārya in his Commentary in the chapter Bhaganādhyāya. He has given us a clue that the numbers of sidereal revolutions of the planets including the node and apogee of the Moon were originally determined by observations though he appeals to Āgama that those numbers were given by Āgama, as accepted and transmitted by Brahmaguptāchārya. Even in the Upapatthis or proofs that Bhāskarāchārya gives regarding the numbers of sidereal revolutions known as Bhagaṇas, we perceive that he consciously commits the logical flaw known as इतरेतराश्रयदोष as we are going to show in that context. The proofs he adduces are indeed based upon Aryabhatāchārya's verse 48. Goḷapāda namely

क्षितिरवियोगाद्दिनकृत्, रवीन्दुयोगात्प्रसाधितश्चेन्दुः
शशितारायोगात् तथैव ताराग्रहाः सर्वे

and on Brahmaguptācharya's verse 12, ch. 20 namely

ज्ञातं कृत्वा मध्यं भूयोऽन्यदिने तदन्तरं भुक्तिः
त्रैराशिकेन भुक्त्या कल्पग्रहमण्डलानयनम्

Aryabhatācharya and his immediate followers Lallācharya and Vateswarācharya make the yuga-pādas of equal duration. Brahmaguptācharya criticises Aryabhata for having said so against the Canons of the Smṛtis as well as Romaka for having ignored the concept of yugas, manvantaras and Kalpa, as this he deems as a heresy (Vide verses 9 & 13 ch. I).

Indeed there seem to be two schools among the ancient Hindu Astronomers one of Brahmagupta who was followed by Sripati, Bhaskara and a number of others and the other of Aryabhata who was followed by Lalla, Vateswara, Bhaskara I, and a host of others mostly hailing from Kerala.

There is an Aryabhata who has been termed Aryabhata II and who was the author of a book named Brihad-Aryabhatiyam or Mahā-Siddhanta as it is also called. M. M. Sudhakara Dwivedi mentions in his Ganaka-Tarangaṇi, that this Aryabhata should have existed after the author of Modern Surya Siddhanta. Aryabhata I, Aryabhata II, many of the Kerala astronomers used a different nomenclature to signify numbers, denoting them by letters. Thus one of the distinguishing features of the Kerala school of astronomers (not all of them) seems to be to use letters for numbers.

One Kalpa = 14 Manvantaras = 14 × 71 yugas + 15 Sandhis in between the Manvantaras each equal to a Krita ie 4 Kalis = 994 yugas + 60 Kalis = 994 + 6 yugas = 1000 yugas = 4320000000 mean Solar years.

Bhaskara speaks of Sandhyas and Sandhyamsas each of them equal to $\frac{1}{12}$ of the yugapādas.

Thus one Kali = 432000 years = 1200 Divyābdas (gods' years each year being equal to 360 Solar years) = 1000 +

100 + 100 Divyābdas since $\frac{1}{12} \times 1200 = 100$ = measure of the Sandhyā and Sandhyamsa times. At this rate Dwapara = 2000 + 200 + 200 Divyābdas and so on. However, the measure of the Sandhyas with respect to a Manvantara does not follow this one-twelfth rule, because a Krita is not $\frac{1}{12}$th of the Manvantara. Thus 1000 Divyābdas = one Kali excluding Sandhyā and Sandhyamsa, whereas 1000 yugas = one Kalpa including the Sandhyas and Sandhyamsas.

When a yuga was conceived as a period wherein the planets make an integral number of revolutions, it goes without saying that they make integral numbers of revolutions in a Manvantara or a Kalpa. When a Kalpa was conceived as the period in which the slow-moving planetary points aphelia and nodes also make an integral number of revolutions, one wonders how a manvantara was conceived. It is further peculiar why such an odd number 71 was chosen, when it was said that 71 yugas make a Manvantara One also wonders why the modern Suryasiddhānta says that after the Kalpa began, creation started only after 47400 Divyābdas whereas neither Brahmagupta nor Bhāskara speaks of this. As a matter of fact Bhāskara mentions later the number of years that had elapsed upto the beginning of the Saka era, as equal to 1972947179, but does not speak when the creation of planets and stars began actually.

Verse 26. Half the life-period of the present Brahma has elapsed; some said that only eight and half years of his life has elapsed — Let the Āgama or tradition be whatsoever; we don't have any need of knowing it because the planetary positions have to be computed only from the beginning of this Kalpa.

Comm. Vateswara it was that mentioned that only eight and half years of the present Brahma had elapsed (Vide verse 10 Madhyādhikara ch. I Vateswara Siddhānta) Vateswara prescribes that Ahargana or the collection of days has to be calculated from the birth-time of this Brahma, but Bhāskara rightly points out that it is a waste of labour, for, all the planets must have returned to the Zero-point of the zodiac ie the beginning of the Star Aswini at the beginning of this Kalpa and hence it is sufficient to calculate only from the beginning of this Kalpa. Further Bhāskara states that when the very planets did not exist during the last elapsed night of Brahma, what is the fun of calculating their positions.

There is also a tradition that there are nine Brahmas and that the present one is the very first. This tradition Bhāskara does not mention, because he exclaims that he does not know how many Brahmas have gone by, Time being without a beginning.

Verse 27. In as much as the creation started only from the beginning of this Kalpa which is the present day-time of Brahma, and because deluge takes place at the end of the day-time, the question of Computing the planetary positions arises only when the planets exist. If some (the allusion is to Vateswara) propose to compute the planetary positions even when the very planets did not exist, may we salute those great people !

Comm. Not necessary.

Verse 28. Six Manus have elapsed in this Kalpa, thereafter twentyseven yugas, as well as three yugapādas namely Kṛita, Tretā and Dwāpara. Further 3179 years of this fourth yugapāda namely Kali have elapsed by the end of the Saka king (which moment was the beginning of the Saka era). Hence in the present Kalpa ie the day-time of this Brahma, 19729 47179 years had elapsed upto the beginning of the Saka era.

Comm. The computation is as follows:

6 Manvantaras = 6 × 71 × 10 Kaliyugas since each Manvantara Consists of 71 yugas and a yuga Consists of

ten Kaliyugas (one yuga = Krita + Tretā + Dwāpara + Kali = 4 + 3 + 2 + 1 = 10 Kaliyugas). The Sandhis that were there in between the Manus and in the beginning of the first Manu are seven and each Sandhi being equal to one Krita or four Kalis, the seven Sandhis = 7 × 4 = 28 Kaliyugas.

Further it is stated that 27 yugas had elapsed in the present seventh Manvantara known as Vaivasvata which are equal to 27 × 10 = 270 Kaliyugas. Further in the present yuga, Krita, Tretā and Dwāpara had elapsed equal to 4 + 3 + 2 = 9 Kaliyugas.

Thereafter in the present Kaliyuga 3179 years elapsed upto the beginning of the Saka era. Thus totalling we have 4260 + 28 + 270 + 9 kalis + 3179 years.

= 4567 kalis + 3179 years.

= 4567 × 432000 + 3179 years.

= 179 294 7179 year as mentioned.

Verse 29. The six Manus that went before the present Vaivasvata were Swāyambhuva, Swārociṣa, Auttama Tāmasa, Raivata and Cakshusha.

Comm. Clear—Upto this point we have seen the Brāhmamāna, the seventh of the nine mānas.

• *Verse* 30. The Sāmhitikas declare that a Samvatsara is equal to the ṭime of a mean sidereal revolution of Guru the Jupiter (This is the Bārhaspatyamāna). The ninth māna named the Manushyamāna is composite of the four mānas (as detailed in verse 31).

Comm. The years of the Bārhaspatya māna are enumerated as Vijaya, Jaya etc which are sixty in number. The same names are also adopted in the cāndramāna ie the luni-Solar reckoning (In vogue in the Andhra Pradesh and some other provinces too) only with the difference that prabhava is taken as the starting year in which sequence Vijaya happens to be the twenty-seventh year. Since the mean sidereal revolution takes roughly 11.8 years; five revolutions take 59 mean solar years. The sixtieth year thereafter of the Jovian Cycle is considered as an Adhi-Samvatsara thereof so that the luni-Solar reckoning as well as the jovian reckoning get wedded together. Thus to get the jovian year we have simply to add 27 to the luni-solar year. The wedding of the two reckonings has its analogy in the process of intercalation which weds the solar reckoning with the luni-Solar only with the difference that months of the latter reckoning are set apart as Adhika or extra months. Just as the process of intercalation brings in its train what is called a Kshayamāsa as per the convention that, that month would be set apart as an Adhikamāsa, which does not carry a Samkrānti ie entrance of the Sun into the next Rasi, in which process there appears a month in which there may occur two Samkrāntis which is hence considered as a Kshayamāsa, just in a similar way that luni-Solar year in which the Jupiter enters the next Rasi, is supposed to be normal whereas that year in which such an entrance does not take place is deemed an Ādhika year and set apart while that year which carries two entrances is deemed as a Kshya year. The process of intercalation which weds together the solar and the luni-solar reckonings will be elucidated further in its appropriate context. The word Sāmhitikas means the authors of the works called Samhitas like the Varāha-Brihatsamhita etc.

Verse 31. The manushya-māna or that which men follow, adopts the year, the Ayana the Ṛtu or the season (six in number during an year) and the yuga according to the movement of the Sun ie according to Sauramāna the months and the thithis according to the luni-solar reckoning ie according to Chāndramāna the Vratas, upavāsas, treatment of diseases, deliveries of ladies, the names of the weeks all these according to the reckoning of civil days ie according to Sāvanamāna and finally the ghatis

to the

Comm. A lunation is divided into thirty tithis, which go by the names pratipat etc. of which there are fifteen in the brighter half of the lunation and fifteen in the dark fortnight. The pratipat tithi is that duration of time beginning from the moment of New Moon and ending when the Moon has over taken the Sun by 12° ; the second tithi named dwitiyā begins at the end of pratipat and lasts upto the point of time when the moon's elongation is 24° and so on. Thus the tithis are seen to be of unequal length in as much as both the Sun and the Moon have unequal motion. The mean duration of a tithi is seen to be a little less than a civil day since a lunation has rongbly 29½ civil days. There is a convention that the tithi which is current at a Sun-rise will be considered to be the tithi of the whole day. As per this convention it so happens that a tithi lasts just a little after Sun-rise and the next tithi vanishes during the same day so that the next but one will be taken for the next day. This vanishing tithi is known as a Kshaya tithi which is also called a Kshayāta. Again it so happens that a tithi may be current at two consecutive Sun-rises beginning a little before the Sun-rise of the first day and extending a little after the next Sun-rise. Such a tithi is called Dina-Traya or a tithi which touches three days. In this matter if seems as though we have gained a tithi but ultinately as a tithi must be less than a civil day, it so happens that on the average there will be a Kshayāha rongbly in 64 tithis. We shall see more about this matter subsequently.

Verse 32. Thus there are nine mānas Mānava, Divya (or of gods), Bārhaspatya (Jovian) paitra, Nākshatra, Saura, Chāndra, Sāvana and Brāhma. But the planetary positions are to be computed by men by their own māna.

Comm. Not necessary.

Here ends the Adhyāya known as Kāla-māna in the **Madhyādhikāra.**

MADHYĀDHIKĀRA – SECTION II
BHAGAṆĀDHYĀYA

Verses 1 to 6. The number of sidereal revolutions of the Sun during a Kalpa is 4320000000. It is also the number of those of Mercury and Venus, and those of the S'ighrocchas of the planets Mars, Jupiter and Saturn.

The Moon makes 57753300000 sidereal revolutions in a Kalpa, the Mars 2296828522, the Mercury's s'ighroccha 17936998984, the Jupiter 364226155, the s'ighroccha of Venus 7022389492 and the Saturn 146567298. The sidereal revolutions of the apogees of the Sun and the Moon and those of the aphelia of Mars, Mercury, Jupiter, Venus and Saturn in a Kalpa are respectively 430, 488105858, 292, 352, 855, 653, 41.

The retrograde sidereal revolutions of the nodes of the orbits of Moon, Mars, Mercury, Jupiter, Venus and Saturn are respectively 232311168, 267, 521, 63, 893, 584.

Comm. Since Mercury and Venus will be oscillating about the Sun in their apparent motion as seen from the earth in a long period of time, the number of sidereal revolutions made by the Sun is also equal to that made by Mercury and Venus. Since, as we see in the Course of the Spaṣṭādhikāra the Sun plays the part of what is called the S'īghroccha of the three major planets named Mars, Jupiter and Saturn, the number of the Sun's sidereal revolutions is also the same as that of their s'ighrocchas.

The reason why the sidereal revolutions of what are called the s'ighrocchas of Mercury and Venus are the same as the heliocentric sidereal revolutions of Mercury and Venus will be clarified in the spaṣṭādhikāra. The reason why the sidereal revolutions of the major planets the same as their heliocentric ones will also be clarified in the same adhikāra.

The reason why we have termed the Mandocchas as apogees in the case of the Sun and the Moon and as aphelia with respect to the other planets is that the Sun moves round the earth relatively while the Moon directly moves round the earth whereas the remainning planets move round the Sun while the Sun moves relatively round the earth. In other words the Sun and Moon have apogees whereas the remaining planets aphelia.

The nodes of the planets are the points of intersection of their orbits with the ecliptic which is the apparent orbit of the Sun. In other words the planetary orbits are inclined to the ecliptic which means that their orbital planes do not coincide with the ecliptic plane.

In Hindu Astronomy the Sun, the Moon and also the two nodes of the lunar orbit which go by the names Rahu and Ketu are also termed as grahas along with the other five which are known as Tāra-grahas or planets resembling stars. The etymology of the word 'planet' is that it moves amongst stars; in this respect the nine Hindu grahas also moving among the stars are eligible to have the same appellation though it is not permitted in modern astronomy. But the word graha has a different connotation etymologically namely गृह्णातीति वा गृह्यते अनेनेति वा ग्रहः ie that which seizes the fates of men is known as a graha. In the course of this work we use the word graha and planet synonymously so that we deem the Sun, the Moon and the nodes of the lunar orbit also as planets.

When the ancient Hindu astronomers knew by observation that the nodes of the lunar orbit have a retrograde motion, and could also measure their mean motion, they extended the analogy to the nodes of the other planets also, whose motion could not be measured during anybody's life-time-Hence the estimate of their mean motion by the Hindu Astronomers naturally went wrong.

Noticing that the Mandoccha of the Moon has a progressive motion which the Hindu astronomers could measure correctly, they extended the analogy to the apogee of the Sun and the aphelia of the other planets whose motion also being very slow could not be measured during the life-time of a man. So here also the estimate of their mean motions of the Sun's apogee and the planetary aphelia went wrong.

Bhaskarācharya has given proofs as to how the sidereal revolutions could be got but as we shall see later in the Spaṣtādhikāra, his proof occasionally suffers from what is called 'इतरेतराश्रयदोष' ie 'begging the question'. We shall however construct our own proofs at that place deferring them for the present, for, the proofs require an elucidation which obtains in the Spaṣtadhikāra alone.

Verse 7. The number of diurnal revolutions of the stars in a Kalpa is 1582236450000.

Comm. In fact this number is that of the diurnal rotations of the earth which is equal to the number of apparent diurnal rotations of the stars, if the earth is deemed as fixed. The only Hindu Astronomer who made bold to say that the earth is rotating and not the stars came under criticism by Brahmagupta and the latter Hindu astronomers. Aryabhata said अनुलोमगति : नौस्थः पश्यत्यचलं विलोमगं यद्वत्, अचलानि भानि तद्वत्. समपश्चिमगानि लङ्कायाम्" ie Even as a man stationed on a moving boat perceives that the trees etc on the banks of the canal, river or lake to be moving in the opposite direction supposing himself stationary, so also men stationed on the surface of the earth (which is like a moving boat) perceive the actually stationary stars to be moving directly from east to west at Lanka ie the equator".

Why none of the latter Hindu astronomers, though many of them could intuit this simple phenomenon, boldly came out asserting this, is rather mysterious. Even today there is such an irrational orthodox type of scholars who are not in touch with modern astronomy, holding the view that the earth does not move.

In Hindu Astronomy it was postulated that there is what is called the pravaha wind, which effects moving of the entire stellar universe along with the planets from east to West. It is a very simple matter to visualize earths' rotation, instead of supposing that the entire stellar Universe is being driven round the earth. The absurdity in this latter supposition might not have been clear to the orthodox type of the Hindu Astronomers, for, they could not measure the dimensions of the giant stars and super-giants, which are everyone of them mighty Suns. Though, however, the dimensions of the Sun were known to them to be far greater than those of the earth, it did not occur to them why a mighty Sun should go round a pigmy earth. Or even if it occured to astronomers like Bhaskara, they dared not to go against the puranic tradition.

Since the stars do not move among themselves while partaking this diurnal motion, the entire starry skies are obliged to go round the earth as a rigid structure, if we suppose that the earth is not rotating about, herself. This kind of supposition is just like a revolving person, revolving about himself and claiming that the entire Universe is revolving round him and not he about himself.

Bhaskara gives the proof of getting this number of diurnal rotations of the stars during an year in the verses 5–7 of Madhyagati Vāsanā, समं भसूर्यावुदितौ etc. as follows. Suppose a star and the Sun rise together today. Tomorrow the star will have arisen earlier than the Sun who will have moved towards the east of the star by his own (apparent) daily motion. So tomorrow's sun-rise will get belated by the duration of time that the arc of the ecliptic covered by the Sun's today's motion takes to rise. This duration of time is variable on two counts; first by the variable motion of the Sun and second by the obliquity of the ecliptic on account of which even equal arcs of the ecliptic will not rise in equal times. In other words the duration of time between two consecutive Sun-rises will not be the same. This duration of a particular day can roughly be calculated by the rule of three as follows. Let the Sun be in a particular Rasi, the rising time T of which could be computed; let the Sun cover an arc of x° in that Rasi on that day. Then the time taken by that arc to rise is $\frac{xT}{30}$, where a Rasi consists of 30°. This time computed in Sidereal measure added to 60 Sideral ghatis is equal to the length of the day. During the course of an year ie the time taken by the Sun to move round the ecliptic starting from the Zero-point of the zodiac and again returning to the same point, the Sun will have made one revolution less than the stars. Thus if 'R' the number of diurnal revolutions of the Sun (where R will be not an integer) during an year or what is the same the number of Sīvanā or civil days during an year, they are equal to R + 1 sideral days. Hence the number of sideral days in a kalpa will be equal to the number of civil days in a kalpa together with the number 4320000.000 which is the number of revolutions made by the Sun relative to the stars.

In this context we are to know the number of civil days in a kalpa. The proof given by Bhaskara in Gaṇitadhyāya under verses 1–6 in Bhagaṇopapatthi is as follows. Draw a circle on a horizontal plane and place a vertical pole called gnomon at the centre of the circle. Observe the point of intersection of the gnomon's shadow with the circumference of the circle at Sun-rise on a day in the Uttarāyaṇa ie during the course of the Sun's north-word journey, just at the time when his rising point is very near east point and also to the south thereof. Then from that day go on counting the number of days, which will be 365 when the Sun again rises very nearly at the same point and just to the south of the east point. It will be found that the Sun will rise the next day just a little to the north of the east point, which means that the Sun has taken 365 days and a fractional part of a day to complete his revolution round the stars. Having noted the two points of intersection of the gnomonic shadow with the circle, on those two consecutive days when the Sun happens to rise just a little to the south and then on the next day just a little north of the east point, and having measured the arcs in minutes between those points of intersection and the western point of the horizontal circle (western because the gnomonic shadow of the rising Sun is cast towards west) then the following rule of three is to be applied. If during 60 ghatis of the day, the sum of the arcs in minutes is covered, what will be the time taken by the shadow to traverse the arc between the west point and the northern point of intersection. This added to 365, gives the number of civil days in an year. Here it will be noted that this year is tropical because rising in the east signifies the Sun's position at an equinox.

Verse 8. The number of solar days in a kalpa is equal to 1555200000000 and of the lunar days or tithis is 1602999000000.

Comm. The solar days here cited are counted at the rate of 360 per year; and the lunar days at the rate of 30 per lunation. Tithi is defined as mentioned by us under verse 31 of the previous section. Since there are thirty tithis in a lunation, their enumeration is quite alright; but there seems to be a little oddity in saying that there are 360 solar days in an year. This kind of a solar day is a little longer than a civil day and does not correspond to any particular motion of the Sun, say for example the time taken by the Sun to move a degree along the ecliptic. definition of solar days pertains only to a stipulation

360 solar days constitute a solar year and no tion is given for a single solar day. This definition of solar days, though apparently artificial, has some significance, namely that the difference of the solar and lunar days defined above constitutes 15933000000 Adhikamāsas in a kalpa at the rate of 30 tithis per month. In other words the difference between the solar and lunar days, is the number of tithis that the luni-solar reckoning gains over the solar. This topic will be dealt with later.

Verse 9. The number of civil days in a kalpa is equal to 1577916450000; the number of the diurnal revolutions of the stars minus the number of sidereal revolutions of any particular planet constitute the days of that particular planet with respect to the earth.

Comm. The civil days in a kalpa are evidently the number of Sun-rises. These are as mentioned before the difference of the number of diurnal revolutions of the stars and the number of the sidereal revolutions of the Sun. By analogy, the number of the days of a particular planet with respect to the earth or what is the same the number of risings of that planet in a kalpa as seen from the earth, is the difference of the number of the diurnal revolutions of the stars and the number of the sidereal revolutions of that planet in a kalpa. Thus we have Saura-Ku-dināni, Chāndra-Ku-dināni, Bhauma-Ku-dināni etc, where the word 'Ku' means the earth. The Saura-Ku-dināni are the civil days defined before. It will be noted that the Chāndra-Ku-dināni are not the lunar days.

Verse 10. The number of Adhikamāsās or intercalary months in a kalpa is equal to 1593300000 and the number of Dina-Kshayas is 25082550000.

Comm. A luni-solar year as per Cāndramāna ie the luni-solar reckoning consists of twelve lunations; as such its length falls short of that of the solar year by 11 days 3 ghatis, 52 Vighatis and 30 Sūkshmaghatis. If both the solar and luni-solar years begin simultaneously this year, by the end of the luni-solar year, it will have gained over the solar year the above-mentioned days. This difference which goes by the name Adhimasa-Sesha or Suddhi accrues to the length of a lunation in about 32½ solar months. Unless this accruing difference is set apart by some device, and the beginnings of the two years again brought together, the luni–Solar year looses its significance of an year, for, it does not accord with seasons. It being a convention that an year should begin with the spring, if the luni–Solar year also is to begin with the spring, it is to be wedded to the solar year by some device. The device adopted in this behalf was to leave out a month in the luni-Solar reckoning as soon as it could be seen that a month has been gained by this reckoning over the solar. This knowledge is had from the following fact. The zodiac is divided into twelve equal portions called Rasi's beginning from the first point of the Hindu Zodiac, ie. from the first point of the asterism division called Aswini. The Sun traverses each of these Rasis in one Solar month, and on account of the unequal motion of the Sun, these solar months are of unequal length. The entrance of the Sun from one Rasi into another is called a Samkrānti, and the moments of Samkrāntis are held to be holy for religious purposes. The solar month being a little longer than a lunation, normally a Samkrānti occurs in a lunation. But it so happens that in a particular lunar month this Samkrānti might not occur. The Suddhi which is the time between the moment of New Moon and the subsequent Samkrānti and which is therefore the time gained by the luni–Solar reckoning over the Solar, having accrued to a lunation, it is an indication that the luni–Solar reckoning has gained a lunation over the Solar. That lunation not carrying a Samkrānti is termed an Adhikamasa and is left out. That it is left out is connoted by the word Adhika which means extra, as well as by the convention that no auspicious celebrations like a marriage etc. should not take during that month. The next lunar month is termed as the Nija-māsa or the month which is the true. In this convention however, there may occur two Samkrāntis in one particular lunar month. For this to happen two criteria are to be satisfied namely that, that particular lunar month must be greater than the concurrent solar month and secondly one of the two Samkrāntis must occur immediately after a New-moon so as to permit the second Samkrānti to occur just before the lapse of the lunar month. The Sun coming to his perigee roughly about the lunar month Mārgasira, and as such having the quickest motion at that point, he covers the length of the Rasi of 30° within the shortest span of time making that Solar month shorter than the corresponding lunar month. Since the months Kārtica and pausha, one before and the other after Mārgasira, also being thus longer than the corresponding solar months, two Sankrāntis could occur, if at all, in these three lunar months. If they occur, that lunar month is termed a Kshayamāsa. Since two lunar months are to correspond to two Samkrāntis, convention has it that two lunar months lapse 'simultaneously' during that lunation, the previous lunation running during the forenoon and the subsequent lunation during the afternoon. Thus a Kshaya māsa is also called a yugalibhūta māsa or a twin māsa, which therefore makes one lunation virtually vanish. On account of this vanishing', it is called a Kshayamāsa. After what an interval of time this Kshaya māsa occurs how a month precedes it having no Samkrānti and how again a lunation follows it without a Samkrānti will be seen in a subsequent context. This convention pertaining to the institution of an Adhikamāsa goes by the name 'Inter-calation'.

It was mentioned before that on an average an Adhikamāsa occurs once in 32½ solar months roughly—Also, Adhikamāsas are the excess of lunations over the solar months in a given period. There being 51840000000 solar months in a Kalpa and 53433300000 lunations the number of Adhikamāsas is therefore 1593300000 as mentioned.

Kshayāhas were explained before and their number in a Kalpa is the difference of the civil days and the tithis.

Verses 11, 12. The numbr of solar months in a Kalpa is 51840000000; the number of lunar months is 53433300000. The number of solar months heing subtracted from the number of lunations, we have the number of Adhikamāsas—The number of solar days together with the days of Adhika months are equal to the lunar days or the tithis; or again the lunar days minus the Kshayāhas are equal to the number of civil days or the reverse will be had by a reverse process.

Comm. Already explained.

Verse 13. The excess of the sidereal revolutions of the moon over the number of the Sun's sidereal revolutions is equal to the number of chāndramāsas or lunations.

Or again the excess of the sum of the sidereal revolutions of the moon and the tithis over the sum of the lunations and the diurnal revolutions of the stars is equal to the number of Kshayāhas.

Comm. The first part is clear, Regarding the second, let the number of lunations be x and the number of the sidereal revolutions of the moon be y. Then $y-x=z$, the number of the sidereal revolutions of the Sun because $y-z=x$ from the first part above. If now, the number of the diurnal revolutions of the stars be t, then $t-z=t-(y-x)=t+x-y=$ civil days. Subtracting these civil days from 'U' the number of Tithis, we have the number of Kshayāhas namely $U-(t+x-y)=(U+y)-(t+x)$ which accords with the statement.

Verse 14. The number of Adhikamāsas is equal to the excess of the number of sidereal revolutions of the Moon over thirteen times the number of sidereal revolutions of the Sun.

Comm. Let x and y be the numbers of sidereal revolutions of the Moon and the Sun respectively. The x—y is evidently the number of lunations as mentioned before. Also 12 y is the number of solar months which if subtracted from the number of lunations will give the number of Adhimāsas ie $x - y - 12y = x - 13y =$ Adhikamāsas as mentioned.

This completes the Bhaganādhyāya of Madhyādhikāra.

GRAHĀNAYANĀDHYĀYA ग्रहानयनाध्यायः

Verse 1. To Compute the Ahargaṇa the collection of days from the beginning of Kalpa ie from the beginning of creation. Multiply the number of sidereal solar years from the beginning of Kalpa by 12; add the number of elapsed lunar months; multiply by thirty; add the number of elapsed tithis. Let this number be x. Then $\left[\frac{x \times A}{s}\right]$ the integral number obtained by dividing the product of x and A the number of Adhikamāsas in a Kalpa by s the number of solar days thereof gives the number of elapsed Adhikamāsas. Multiply this number of Adhikamāsas by 30 and add to x. The result gives the number of elapsed tithis, Let this be y. Then $\left[\frac{y \times K}{T}\right]$ the integral number obtained by dividing the product of y and K the number of Kshayāhas in a Kalpa by T the number of Tithis in a Kalpa, gives the number of Kshayāhas. Subtracting this number from y, we have the Ahargaṇa ie the number of the elapsed civil days from the beginning of Kalpa. This Ahargaṇa has its beginning on Sunday and is itself constituted of mean solar days. While computing the Adhikamāsas or Kshayāhas, the integral numbers of the quosients alone should be taken rejecting the remainders.

Comm. The elapsed number of solar years is directed to be multiplied by 12 in the beginning. This number does not constitute purely solar years. Solarity was secured upto the point of the last intercalation of an Adhikamāsa and thereafter one or two luni-solar years would have been added. But Construing these one or two luni-solar years as mean solar years does not make a difference while computing the Adhikamāsas, for the following reason. Adhikamāsas normally occur once in three years. Even supposing that the number of the elapsed years contain three luni-solar years after the year carrying the last intercalary month, the error in construing them to be solar will be roughly minus one solar month. In other words the difference between three mean solar years and three luni-solar years will be roughly one solar month. Since one Adhikamāsa occurs roughly in $32\frac{1}{2}$ solar months, so the number of Adhikamasas obtained by construing three luni-solar years as mean Solar years will be in default by roughly $\frac{1}{32\frac{1}{2}}$ or $\frac{2}{63}$. As we are counting only the integral number of Adhikamāsas obtained as a quotient rejecting the remainder, the above default of $\frac{2}{63}$ should not normally effect the quotient. Most rarely, however, it might effect the quotient by one, for which provision is made by Bhaskara in verse 3 under Adhimāsādinirṇaya section, Madhyamādhikara namely स्पष्टोऽधिमास: पतितोप्यलब्ध: etc. which means that if the quotient is in default by one, where it is definitely known that one more Adhikamāsa did occur, we are directed to add one and if we certainly know that the quotient contains one more Adhikamāsa, when the Adhikamāsa is shortly to occur and has not occrued we are directed to substract one from the quotient. This principle of सैकत्वनिरेकत्व is to be observed even in the context of Kshayāhas ie we are directed to add one or subtract one from the number of civil days by adjusting the Ahargaṇa to the week-day on which the Ahargaṇa is sought to be found.

Thus construing the elapsed number of years to be solar, multiplying them by twelve we have the elapsed solar months. Adding to this the number of the elapsed luni–Solar months construing these to be solar, which fact also does not affect normally the number of Adhikamāsas to be obtained (for the same reason above) and multiplying by 30 and adding the elapsed tithis we have the elapsed number of solar days (This number may not be exactly the elapsed number of solar days, for, we have construed the one, two or three luni-Solar years as mean solar as well as the elapsed lunations of the present year as solar months; but this difference does not affect the computation of Adhikamāsas as mentioned above). Then, as we are given that 15933,00000 Adhikamāsas occur in 1555200000000 solar days of the Kalpa, if x be the number of solar days found

give the number of the

Adhikamāsas. Adding these Adhikamāsas multiplied by 30, to x the solar days obtained above, we have the Tithis elapsed upto the day in question. If now, we subtract the Kshayāhās from these Tithis, we shall have the number of Sāvanāhas or the Ahargaṇa required. Since in 1602999000000 Tithis of the Kalpa there will be 25082550000 Kshayāhas, if y be the Tithis above obtained, $\frac{y \times 25082550000}{1602999000000}$ will be the Kshayāhās from the beginning of the Kalpa upto the day in question; subtracting these from y, we have the required Ahargaṇa.

Verse 4. Computation of the planetary positions.

The Ahargaṇa multiplied by the number of sidereal revolutions of a planet and divided by the number of civil days in a Kalpa gives the planet ie its number of revolutions upto the day concerned both integral and fractional.

Comm. Applying 'Rule of three', if in C the number of civil days in the Kalpa, the planet makes P sidereal revolutions how many revolutions would have been made in A the Ahargaṇa? The answer is $\frac{A \times P}{C}$. In this, the integral quotient gives the number of complete revolutions made; the remainder, multiplied by 12 and divided by C again, gives the number of Rasis covered by the planet from the Zero-point of the Zodiac, and again the remainder by 30 and divided by C gives the number of degrees covered in the next Rasi; proceeding thus, the planetary position could be had next in minutes and then in seconds of arc also. This position is the mean position of the planet, at the time when the mean Sun is very nearly at the Eastern horizon at Lanka. Why it is said "very nearly at the horizon" will be clear in the context of udayāntara Samskāra to be dealt with later in Spaṣṭādhikāra. To obtain the planetary position at the moment when the mean Sun is exactly on the horizon, we have to apply what is called the Udayāntara correction and again to obtain the position at the moment of True Sun-rise we have to apply what is called the Bhujāntara correction, both of which will be dealt with in Spaṣṭādhikara. Having thus got the mean planetary position at True Sun-rise, we have to apply one or two as the case may be, corrections to obtain the True planetary position, besides effecting two more corrections known as Desāntara and chara to obtain the True planetary position at the time of True Sun-rise not at Lanka but at the place concerned.

Verse 5. To obtain the position of the mean Moon, when the mean Sun is known from what is called Avama-Seṣa.

The Avama Seṣa divided by 131490000000 in degrees is to be added to twelve times the elapsed tithis and the result added to the Sun's position gives the position of the Moon. Conversely the position of the Sun can be had from the position of the Moon.

Comm. The moon's longitude minus the Sun's longitude known as elongation is called Vyarkēndu, which divided by twelve gives the number of elapsed tithis; a lunation which is the time in which the Moon overtakes the Sun by 360°, contains thirty tithis and a tithi is defined as the time, in which the Moon overtakes the Sun by 12°, beginning from the moment of conjunction ie Amāvāsyā. Hence if the Sun's longitude be x°, aud y be the number of elapsed tithis, integral or fractional $(x + 12y)^\circ$ will be the longitude of the Moon. If, however, we consider y only as the integral number of the elapsed tithis, we will have obtained the Moon's position from the formula $(x + 12y)^\circ$ at the ending moment of the tithi on the previous day. The time in between this ending moment of the tithi and the Sun-rise of the day concerned is known as Avama-Sesha, since the Avama-days or Kshayāhas are the difference of days between the number of tithis in a given period and the number of civil days during the same period. In other words, the excess of a civil day over a tithi which falls short of it, is the part of a civil day that contributes towards the number of Kshayāhās in a given period. Thus we have to compute the increase in the Moon's longitude during the aforesaid Avama-Sesha to obtain his longitude at the Sun-rise of the day concerned. But this Avama-Sesha is of the form $F\left\{\frac{t \times K}{T}\right\}$ where 't' is the number of elapsed tithis upto the Sun-rise of the day concerned K the number of Kshayāhas in a Kalpa, T is the number of tithis in a Kalpa and $F\left\{\frac{t \times K}{T}\right\}$ signifies the fractional part called Avama-Sesha, the integral quotient having given the number of elapsed Kṣhayāhas. Hence writing $F\left\{\frac{t \times K}{T}\right\}$ as $\frac{R}{T}$ where R is the remainder obtained by dividing $(t \times K)$ by T, the Avama-Sesha is of the form $\frac{R}{T}$. This Avama-Sesha being mean solar, it has to be rendered luni-Solar, which process is known as स्चाद्रीकरणम्. The rule of three used in this behalf is 'If for C civil days we have T tithis of the Kalpa, what shall we have for $\frac{R''}{T}$? The answer is $\frac{R}{T} \times \frac{T}{C} = \frac{R}{C}$. This then is the balance fractional part of a tithi that is there in between the end of the elapsed tithi and the Sun-rise of the day concerned. This part of a tithi is to be multiplied by

12 to give in degrees the increase of Moon's longitude from the ending moment of the tithi on the previous day. Thus this increase is $\frac{R}{C} \times 12$; but C = 1577916450000

$$\frac{R}{C} \quad 12 = \frac{1577916450000}{12} = 131490000000$$

approximately.

Thus this increase is to be added to (x + 12 y)° where x° is the longitude of the Sun and y the integral part of the elapsed tithis so that the Moon's longitude is

$$12\ y)^{\circ} \qquad \overline{131490000000}$$

Verses 6, 7. Computation of the positions of the Sun and the Moon from the Adhimāsa-Sesha and Avama-Sesha. The Avama-Sesha divided by 27110000000 is termed an additive constant in minutes of arc to the Sun's position ; the same Avama-Sesha multiplied by 13 and divided by 35 is termed such an additive constant to the position of the Moon ; Construe that the Sun's position is given by as many degrees as there are elapsed tithis after the beginning of Chaitra and that the Moon's position is given by Thirteen times the same. Let these positions of the Sun and the Moon be diminished by a number of degrees equal to what is obtained by dividing the Adhimāsa-Sesha by the number of lunations in a Kalpa. Then add the respective additive constants to the positions of the Sun and the Moon so obtained. The results will be the positions of the Mean Sun and the Mean Moon.

Comm. Here the data are the Adhimāsa-Sesha and the Avama-Sesha and nothing else and the problem set is to find the Mean positions of the Sun and Moon. The Adhimāsa-Sesha is of the form $\frac{R' \times 30}{S}$ where R′ is the remainder obtained while finding the elapsed Adhimāsas by dividing the product of the number of lunations in a Kalpa and (12 x + y) 30 + t where x is the number of elapsed years, y the number of elapsed lunations t the number of elapsed tithis in the current lunar month at the time when the Ahargaṇa is being computed and S the number of Solar days in a Kalpa. The form of the Avama-Sesha was formerly stated as $\frac{R}{T}$. The procedure given is as follows. In the first place, we are asked to assume that the number of elapsed solar days is equal to the number of elapsed tithis from the beginning of Chaitra ie the beginning of the luni-Solar year, since we do not know when the Solar year began; so, at the rate of 1° per a Solar day (A Solar day is not a mean solar day, Vide definition given before under verse 8. Bhagaṇādhyāya) the Sun's position is given by as many degrees as there are elapsed tithis; and the Moon's position must be 13 times the same since each tithi means an increase of 12° in the elougation and if x° be the Sun's position, the Moon's position must be 12 x° ahead of the Sun ie 13 x°. Having got thus approximate positions of the Sun and the Moon, we have to make amends for the roughness of the assumption made. In assuming that the longitude of the Sun is equal to the number of elapsed tithis, we have overestimated the longitude since it has to begin from the beginning of the solar year. The time in between the beginnings of the luni-Solar year and the solar year is known as the Adhimāsa-Sesha at the time of the beginning of the solar year, measured in tithis; also, we have committed an error in assuming the Sun's rate to be 1° per tithi. This also is due to Adhimāsa-Sesha subsequent to the beginning of the Solar year upto the day concerned. Thus the entire error committed by assuming the Sun's longitude to be as many degrees as there are elapsed tithis from the beginning of the luni-Solar year. is no other than the Adhimāsa-Sesha at the day concerned, But this Adhimāsa-Sesha is of the form $\frac{R' \times 30}{S}$ as mentioned and is in tithis.

This has to be converted into solar days to give us the number of degrees of the error. The conversion is effected by the rule of three as "If T tithis of a Kalpa constitute S solar days of the Kalpa, what number of solar days corresponds to $\frac{R' \times 30}{S}$?" The answer is $\frac{R' \times 30}{S} \times \frac{S}{T} = \frac{R' \times 30}{T} = \frac{R'}{T/30} = \frac{R'}{L}$ where L is the number of lunations in a Kalpa. This $\frac{R'}{L}$ being the number of solar days corresponding to the Adhimāsa-Sesha upto the day concerned, the corresponding longitude of the Sun namely $\frac{R'^{\circ}}{L}$ must be subtracted from the Sun's longitude. Now the question arises whether we have to multiply this $\frac{R''^{\circ}}{L}$ by 13 to be subtracted from the longitude of the Moon; not necessary, enough to subtract $\frac{R'^{\circ}}{L}$ only, for, the Adhimāsa-sesha extends from the preceeding Amāvāsya only when the Moon's longitude was equal to the Sun's longitude. We have thus got the mean positions of the Sun and the Moon at the ending moment of the tithi on the previous day. To get their mean positions at the Sun-rise of the concerned day, we have to make amends for the time in between, which is no other than Avama-Sesha as a fraction of a mean solar day. Here Bhaskara makes an ingenious approximation. The maximum Avama-Sesha could be a tithi only and the Sun moves roughly by his daily mean motion during a tithi. So, the rule of three adopted is 'If for one tithi, the Sun's daily motion is to be reckoned, what for the Avama-Sesha? The answer is Avama-Sesha multiplied by the Sun's daily motion. The Avama-Sesha being of the form $\frac{R}{T}$, $\frac{R}{T} \times m' = \frac{R}{T} \times 59\frac{8'}{60}$

$$\frac{\frac{R'}{T}}{59\frac{2}{15}} = \frac{R'}{\frac{1602999000000 \times 15}{887}} = \frac{R'}{27110000000} \text{ very}$$

approximately.

This must be added to the position of the Sun to get his position at the Sun-rise concerned. In the case of the Moon the above additive constant of the Sun, is to be multiplied by $13\frac{13}{35}$ because the Moon's daily motion is so many times that of the Sun. Hence the additive constant in the case of the Moon is $x \times 13\frac{13}{35} = 13\,x\,(1 + \frac{1}{35})$ where x is the additive constant in the case of the Sun. This agrees with what Bhaskara has stated.

Verses 8, 9. Another way of computing the planetary positions.

The mean position of the Sun in Rasis minus $\frac{A \times G}{131493037500}$ Rasis where A stands for the Ahargaṇa, G stands for the Sāvana days of the planet concerned in a Kalpa gives the position of the planet in Rasis. Let pandits find out other similar methods.

Comm. The Sāvana days of a planet is the excess of the diurnal rotations of the stars over the sidereal revolutions of the planet ie $D - P = G$ where G stands for the Sāvana days of the planet. Hence $P = D - G \therefore \frac{A \times P}{M}$ where A is the Ahargaṇa, and M the number of mean solar days in a Kalpa $= \frac{AD}{M} - \frac{AG}{M}$. But $\frac{A \times P}{M} = I + F$ where I is the integral number of the revolutions of the planet and F the fractional part of a revolution which is the planetary position required. Thus from the equation $\frac{A \times P}{M} = \frac{A \times D}{M} - \frac{A \times G}{M} = I + F$. Omitting the integral part in $\frac{A \times D}{M}$ and signifying the remainder as f^1, we have $f' - \frac{A \times G}{M} = I' + F$ where I' is some integer other than I. But $\frac{A \times D}{M}$ which gives us the number of diurnal rotations of the stars upto the day concerned is equal to the Ahargaṇa plus the number of revolutions of the Sun upto the day concerned, because Ahargaṇa + Revolutions of the Sun = diurnal rotations of the stars upto the day concerned. Omitting the integral Ahargaṇa and the integral number of revolutions of the Sun we have that the fractional part of $\frac{A \times D}{M}$ ie f' is no other than the Sun's position. Hence.

$$\frac{A \times P}{M} = I + F = f' - \frac{A \times G}{M} \text{ and}$$

$\frac{A \times G}{1577916450000}$ revolutions $= \frac{12\ R}{1577916450000}$ Rasis $= \frac{R}{131493037500}$ Rasis. Thus the planetary position is equal to the Sun's position minus $\frac{A \times G}{131493037500}$ where the integral parts on either side could be ignored.

Here there is a peculiarity in this method. The planet could right away be obtained from the formula $\frac{A \times P}{M}$ where A is the Ahargaṇa, P the number of sidereal revolutions of the planet and M the number of mean Solar days in a Kalpa. Though the given method is more cumbrous than finding through the above formula, Bhaskara deliberately gives it to show the equivalence of various procedures at the same time giving us a beautiful technique as mentioned in his commentary. Thus in the above equation $\frac{A \times P}{M} = \frac{A \times D}{M} - \frac{A \times G}{M}$ the first term on the right hand side could be termed the Bha-bhrama-graha and the second term $\frac{A \times G}{M}$ the graha-Sāvana-Dina-graha. We have seen above that the first term is no other than the Mean Sun ignoring the integral number.

Verses 10, 11. Proof of other methods of computing planetary position. Even as the sums or differences of two or more of the numbers of Adhimāsas, Kshayāhas, lunations etc give the number of sidereal revolutions of the planets the sums or differences of two or more of the positions of the imaginary planets which go by the names Adhimāsa-graha, Kshayāhagraha etc computed out of the numbers of those Adhimāsas Kshayāhas etc. give the respective planetary positions.

Comm. This interesting concept is based upon the following principle. Suppose P to be the number of sidereal revolutions of a planet; then $\frac{A \times P}{M}$ gives the planetary position, where A is the Ahargaṇa, and M the number of mean solar days in a Kalpa. Now suppose $P = x \pm y \pm z$ where x, y, z are the numbers of Adhimāsās etc in a Kalpa, then

$$\frac{A \times P}{M} = \frac{A \times x}{M} \pm \frac{A \times y}{M} \pm \frac{A \times z}{M}.$$

The terms on the right-hand-side may be construed to be the Adhimāsa-graha etc, which are the positions of imaginary planets and their sums or differences give therefore the position of the planet, as could be seen from the above equation. Thus for example, we have the equation $P_1 - 13P_2 = a$ where P_1 is the number of the sidereal revolutions of the Moon and P_2 the number of the sidereal revolutions of the Sun, *a* stands for the Adhimāsas because Chāndramasas − Sauramāsas = Adhimāsas; but Chāndramāsas = $P_1 - P_2$ and Sauramāsas = $12 P_2$ so that $P_1 - P_2 - 12P_2 = a$ ie $P_1 - 13P_2 = a$. From this equation

$$A \times P_1 = a \times A + 13P_2 \times A$$

The first term on the right hand side is termed as the Adhimāsa-graha, and the second term is evidently 13 times the position of the Sun whereas the term on the left-hand-side is the Moon's position. Hence, ignoring the integral number of revolutions, we have Moon's position = Adhimāsa – graha + 13 times Sun's position (Ignoring the number of integral revolutions means subtracting integral revolutions or adding them if necessary).

Verses 12, 13. A few more examples on the aforesaid principle. The planetary position obtained by the sum of the sidereal revolutions of two planets, added to or Subtracted from another planetary position obtained by the difference of the sidereal revolutions of two planets and divided by two gives the positions of the two planets respectively, the quicker of the two in the first case and the slower in the second. Similarly the planetary position computed from the difference of the sidereal revolutions of two planets subtracted from the planetary position of the quicker of the two gives the position of the slower whereas the former planetary position added to the position of the slower gives the quicker.

Comm. We have $\frac{(P_1 + P_2) + (P_1 - P_2)}{2} = P_1$ (1) and $\frac{(P_1 + P_2) - (P_1 - P_2)}{2} = P_2$ (2) where P_1 is the number of sidereal revolutions of a quick-moving planet and P_2 that of a slow-moving one. Multiplying the above equations by $\frac{A}{M}$ with the former notation,

$$\frac{\frac{A}{M}(P_1 + P_2) + \frac{A}{M}(P_1 - P_2)}{2} = \frac{P_1 \times A}{M} \quad (3) \text{ and}$$

$$\frac{\frac{A}{M}(P_1 + P_2) - \frac{A}{M}(P_1 - P_2)}{2} = \frac{A}{M} \times P_2 \quad (4)$$

Equations (3) and (4) mean what has been stated in verse (12). Again we have the equations $P_1 - (P_1 - P_2) = P_2$

and $P_2 + (P_1 - P_2) = P_1$ where P_1 and P_2 are the numbers of sidereal revolutions of a quick and slow moving planets respectively. Following the same principle as above we could obtain their positions by multiplying the equations through out by $\frac{A}{M}$ and calling planets on the left-hand-side as (1) Dwiparyayāntarodbhava-graha subtracted from the quick-moving one and (2) the slow-moving planet increased by the Dwiparyayāntara-graha respectively.

Verse 14. The difference of the S'ighra and S'ighra-Kendra as well as the Sum of the Mandoccha and the Manda Kendra give the planet to be computed. Or again a computed planet multiplied by the number of sidereal revolutions of a planet to be computed and divided by the number of sidereal revolutions of the computed gives the planet to be computed.

Comm. $U_1 - P = K_1$ and $P - U_2 = K_2$ where U_1, P, U_2, K_1 and K_2 are respectively the number of Sidereal revolutions of the S'ighroccha, planet, the Mandoccha, the S'ighra-Kendra and the Manda Kendra; hence, we have

$P = U_1 - K_1 = U_2 + K_2$ from these equations also by multiplying througout by $\frac{A}{M}$, we have the planetary position as the difference of the S'īghroccha-graha and S'ighra Kendra-graha or the sum of the Mandoccha-graha and Manda-Kendra-graha.

Again, if P_1, P_2 be the numbers of sidereal revolutions of a computed planet and one to be computed respectively and if p_1, p_2 be the computed planet and the one to be computed, then

$$P_1 \times \frac{A}{M} = p_1,\ P_2 \times \frac{A}{M} = p_2 \text{ so that } \frac{P_1}{P_2} = \frac{p_1}{p_2}$$

$\therefore\ \frac{P_1 \times p_2}{p_1} = P_2$ which means that the position of the planet to be computed is got by multiplying the planetary position of a planet computed by the number of sidereal revolutions of the planet to be computed and dividing by the number of sidereal revolutions of the planet computed.

Verse 15. We get the Ahargaṇa by multiplying the planetary position given in number of revolutions and fraction of a revolution by the number of days in a Kalpa and dividing by the number of sidereal revolutions in a Kalpa. How by indeterminate analysis we get the same Ahargaṇa, given the number of past sidereal revolutions alone, or by the fractional part of a revolution alone, or by the sum of the fractional parts in the case of more items involved, I shall tell later.

Comm. While computing the planet we have the formula $\frac{A \times P}{M} = p$ where A = Ahargaṇa, P = the number of sidereal revolutions of the planet, M = number of mean solar days in a Kalpa and p = the planetary position consisting of the number of past revolutions and also the fraction of a revolution. From the above equation, the Ahargaṇa $A = \frac{M \times p}{P}$ as stated. In tne case of only the integral number of revolutions or the fraction of a revolution alone being given, or the sum of remainders if more items than one are involved, the method of fiinding the Ahargaṇa is illustrated in goḷādhyāya under praśna-adhyāya under verses 12—21.

Verses 16, 17. Method of getting the time in solar years that has elapsed from the beginning of the Kalpa, given the Ahargaṇa.

The given Ahargaṇa multiplied by the number of Kshaya-tithis in a Kalpa and divided by the number of civil days in a Kalpa gives the number of the elapsed Kshaya tithis. Adding these to the Ahargaṇa we have the lunar days L. These again multiplied by the number of Adhikamasas in a Kalpa and divided by the tithis in a Kalpa gives the elapsed number of Adhikamāsas. Multiplying this number by thirty and subtracting from the above lunar days L, we have the elapsed solar days. Dividing these by thirty, we have the number of elapsed solar months, the remainder being solar days. Dividing the solar months by 12, we have the elapsed solar years and the remainder here are the solar months. Thus we have the solar years, solar months and solar days corresponding to the given Ahargaṇa.

Comm. The inverse process detailed here is quite clear.

Verse 18. Computation of the Ahargaṇa and the planctary positions from the beginning of the Kaliyuga.

Find the Ahargaṇa from the beginning of the Kaliyuga either (according to the method described formerly with respect to a Kalpa) and this Ahargaṇa begins from Friday, Computing the mean planetary positions from this Āhargaṇa and adding to their mean positions at the beginning of the Kali which are known as Dhruvakas, we have their planetary positions for the day concerned.

• *Verses* 19, 20. The Dhruvakas of the planetary positions at the beginning of Kali, given in a tabular form.

Mars	Mercury	Jupiter	Venus	Saturn	Solar Apogee	Lunar Apogee	Ascending lunar node	
11 R	11 R	11 R	11 R	11 R	2 R	4 R	5 R	Rasis
29°	27°	29°	28°	28°	17°	5°	3°	Degrees
3′	24′	27′	42′	46′	45′	29′	12′	minutes
50″	29″	36″	14″	34″	36″	46″	58″	Seconds

Comm. The mean Sun and the mean Moon are taken to be in conjunction at the zero-point of the Zodiac. The planetary positions given above are accepted by Bhaskara on the authority of Brahmagupta. The fact that these positions differ from those given by Aryabhata signifies that Brahmagupta observed the True positions in his own time and to obtain those positions by calculation, he must have changed the fundamental constants such as the number of civil days, and sidereal revolutions of planets etc in a Kalpa. Here ends the section known as grahā-nayana.

MADHYĀDHIKĀRA - KAKSHĀDHYĀYA

Verses 1, 2. The circumference of Akāsa—Kakshā.

Astronomers say that the circumference of Akāśa-Kaksha is 18712069200000000 yojanas. Some say it is the circumference of the universe whereas some say that it is the circumference of the mountain which goes by the name Lokāloka. Those who perceive the celestial sphere as a fruit of the emblic myrobalan, (Known as Āmalaka in Sanskrit) placed in the palm, say that it is the circumference of the sphere of solar radiation ie the imaginary sphere whose volume is filled by solar light.

Comm. Bhāskara, in the course of the Commentary makes it clear that he does not subscribe to this idea which is only mythological. Look at his words which are significant and testify to his rational outlook "नाऽस्माकं मतमित्यर्थः, प्रमाण शून्यत्वात" ie "This is not our view; because it is baseless". A yojana will be seen to be equal to 5 miles approximately.

Verse 3. He gives his personal view as follows.

The universe may be bounded or unbounded; our view is that this dimension of the circumference is no other than the distance covered by each planet in the Kalpa.

Comm. This was an assumption made by the ancient Hindu Astronomers, as well as another assumption that the distance covered by every planet during a day is the same. This we shall see later.

Verse 4. The circumference of the universe given above divided by the number of the sidereal revolutions in a Kalpa of any planet gives the circumference of the planetary orbit, so that in a Kalpa, the total distance covered is the circumference of the universe.

Comm. Clear.

Verse 5. The circumferences of the orbits of the Sun, Moon and the Stars.

The circumference of the Sun's orbit is $4331497\frac{1}{2}$ yojanas, that of the Moon 324000 yojanas, of the stellar sphere 259889850 yojanas.

Comm. Later, we are told by Bhaskara that the circumference of the earth is 4967 yojanas and its diameter is 1581. As the method given by him in the Commentary in that context, to measure the circumference of the earth is correct, we may take it that the ancient Hindu Astronomers could estimate the same correctly. If that be so, when Bhaskara gives the circumference to be 4967 yojanas, it means 4967 yoj $= \frac{3960 \times 44}{7}$ miles or 1 yojana

$= \frac{3960 \times 44}{7 \times 4967} = 5.01$ miles or what is the same 1581 yojanas $= 7920$ miles ie one yojana $= 5.01$ miles approximately. With this measure of a yojana, the Moon's mean distance from the earth's centre should be (as given above)

$$\frac{324000 \times 7}{44} \times 5 \text{ miles} = 257725 \text{ miles approximately.}$$

This seems to be a fair estimate and we have to find out how this estimate could be made—Indeed, there are many elementary trigonometrical methods of finding the distance of the Moon. Which of them was used by the Hindu Astronomers, we have to discuss. Bhaskara, however takes it implicitly from Brahmagupta's version. The latter does not mention from what source he derived it but simply mentions that he has resuscitated the Brahma–Siddhānta, which grew obsolete. Either he or the author of Brahma–Siddhānta must have computed this distance using trigonometry. The following seems to be the simplest method, by which the Moon's distance was originally estimated–Refer fig. 1. Let z be the zenith–distance of the Moon as obser-

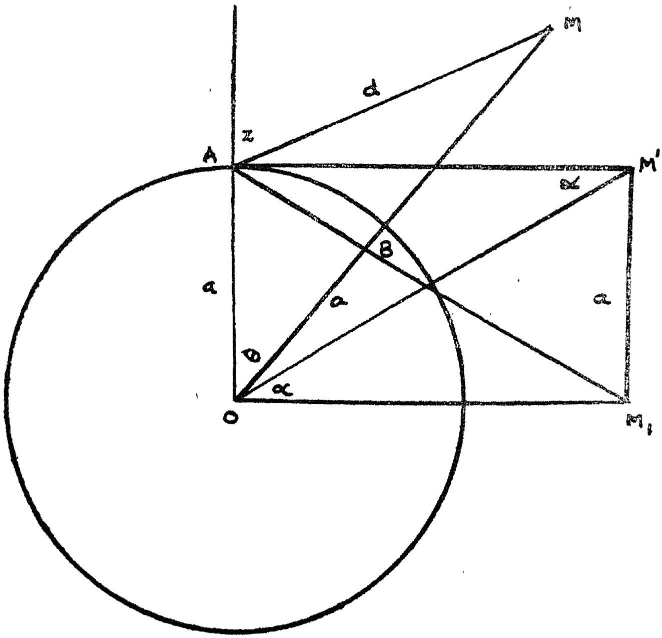


Fig. 1

ved from a place A on the primary meridian going through Lanka, Ujjain, Kerukshetra etc by an instrumeut (A protractor) described by Bhaskara under verse 5 Chandragrahaṇādhikāra, grahagaṇita at the time of transitting. Let B be a sublunar point on the earth at the moment of that observation, where B is also on the same primary meridian. Since both the places happen to be on the primary meridian, and such places were primarily known, both the observations could be made simultaneously. Knowing the distance A B between the two places, the angle θ subtended by A B could be easily got from the triangle A O M where O is the earth's centre and M the Moon. As a first approximation, $\frac{d}{\theta} = \frac{d + a}{\text{Sin } Z} = \frac{a}{\text{Sin } Z - \text{Sin } \theta}$ taking O M roughly to be equal to (a + d). In the above equation, a being known

$d = \dfrac{a \operatorname{Sin} \theta}{\operatorname{Sin} Z - (\operatorname{Sin} \theta)}$. Strictly speaking $\dfrac{d}{\operatorname{Sin} \theta} = \dfrac{a}{\operatorname{Sin} (Z\text{-}\theta)}$

so that $d = \dfrac{a \operatorname{Sin} \theta}{\operatorname{Sin} (Z\text{-}\theta)}$. Since $(\operatorname{Sin} Z - \operatorname{Sin} \theta) < \operatorname{Sin} (Z\text{-}\theta)$

$\therefore$ the estimate of d obtained is a little greater than its true value. We shall discuss other possible methods of finding the Moon's distance in the chapter on lunar eclipses.

Having got the distance of the Moon as afore-said, it was easy to obtain the angle $\propto$ marked in fig. 1 for,

$$\operatorname{Sin} \propto = \frac{M_1 M^1}{O M^1} = \frac{a}{d} = \frac{2\pi a}{2\pi d} = \frac{4967}{324000}$$

Since $\propto$ is small $\operatorname{Sin} \propto = \propto$ radians.

$$\therefore \propto = \frac{4967 \times 3438}{324000} = \frac{4967 \times 191}{18000} = 52.7'$$

Since the Moon's daily average motion $790' - 35''$ is had in sixty Nadis, $52.7'$ of motion is covered in $\dfrac{52.7 \times 60}{\;}$ approximately $= \frac{60}{15} = 4$ Nadis. Thus the fact that we are given the horizontal parallax as 4 Nadis in the context of lunar eclipses is based on this.

Having obtained thus the distance of the Moon from the centre of the earth ε, and having measured the angular diameter of the Moon's disc with the help of the protractor mentioned above, from the triangle εAM (Ref. fig. 2) where

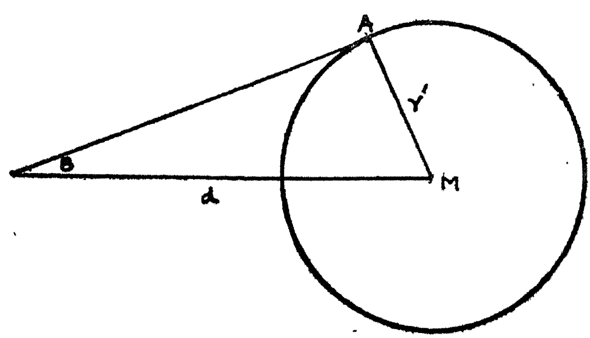


Fig. 2

εA is a tangent to the Moon's disc and M the centre of the Moon, Sin B $= \frac{r'}{d}$ where B is the angular semi-diameter of the Moon's disc, r' = the spherical radius of the Moon's disc measured in yojanas, so that since Sin B$=$B$'$ in Hindu trigonometry when the angle is small,

$$324000 \qquad 3438 \qquad \therefore 2\pi r' = \frac{324000 \times 16}{3438}\ ; \text{ hence}$$

$$r' = \frac{18000 \times 16}{191} \times \frac{7}{44} = \frac{504000}{2101} = 240 \text{ yojanas.}$$

Bhaskara gives the semi-diameter of the Moon's disc to be $16'\text{-}0''\text{-}9'''$ so that the above r' is very approximately 240 yojanas as given by Bhaskara.

Now with these constants pertaining to the Moon's distance, and his spherical radius, it was sought to find the spatial distance traversed by the Moon during a day. The rule of three used was " If $16'\text{-}0''\text{-}19'''$ of the Moon's angular semi-diameter pertains to a spatial distance of 240 yojanas at his orbit, what does the mean daily motion of $790'\text{-}35''$ pertain to ? " The answer is $\frac{683064000}{57609} = 11858\frac{3}{4}$ yojanas as given by Bhaskara elsewhere or taking the Moon's mean semi-angular diameter to be $16'$ only, and using the rule of three " If $16'$ at the lunar orbit correspond to 240 yojanas, what do $790'\text{-}35''$ correspond to " we have

$$\frac{790'\text{-}35'' \times 240}{16'} = 15 \times 790\tfrac{7}{12} = \tfrac{5}{4} \times 9487 = 9487 +$$

$= 11858\frac{3}{4}$ yojanas as given by Bhaskara.

Thus having obtained the daily spatial motion, the Hindu Astronomers assumed all the other planets (including the Sun) to have the same daily mean spatial motion. On this assumption since the, Moon's orbit will be $27.3217 \times 11858\frac{3}{4} = 324000$ yojanas, and the circumference of the universe will be $324000 \times 57753300000$ $= 18712069200000000$ yojanas the Sun's orbit will be

$$\frac{\text{circumference of the universe}}{\text{Number of sidereal revolutions}} = 4331497\tfrac{1}{2} \text{ yojanas}$$

$$\left(\text{because } \frac{324000 \times 57753300000}{4320000000} = \frac{3240}{432} \times 577533\right.$$

$$\left. = \frac{15}{2} \times 577533 = \frac{8662995}{2} = 4331497\tfrac{1}{2}\right).$$

Also, the circumference of the stellar universe was presumed to be sixty times that of the Sun's orbit.

With the constants obtained for the Moon's diameter and distance and with assumption that all the planets have the same daily motion, the constants pertaining to the Sun and the other planets were obtained. We shall resume this topic in another context.

Verse 6. The mean daily motion of the planets.

The circumference of the universe divided by the number of days in the Kalpa, gives the daily spatial motion of a planet. The planets move thus a distance of $11858\frac{3}{4}$ yojanas in a day.

Comm. Already explained.

Verse 7, *and half the verse* 8. The Ahargaṇa multiplied by 11859 decreased by the quotient obtained by dividing the product of the Ahargaṇa and 9921 by 35419 gives the distance covered by a planet in yojanas. These yojanas divided by the circumference of the planet's orbit gives the fraction of a revolution and the integral number of revolutions made.

Comm. Let the Ahargaṇa be A; every planet should have described a space equal to A×D where D is the distance traversed per day and is just a little less than 11859, or more correctly should have described a space equal to $A \times \frac{C}{c}$ where C is the afore-said circumference of the universe, and c the number of civil days or mean solar days in a Kalpa. For the sake of an easy computation Bhaskara gives here an interpolation. In the first place, we are asked to multiply A by 11859 by which, an excess is there in the result; then this excess is sought to be removed. The excess is $A \times 11859 - \frac{A \times C}{c}$ because $A \times \frac{C}{c}$ is the correct distance described. In other words, the distance described by any planet is

$$\frac{A \times C}{c} = A \times 11859 - \left(A \times 11859 - \frac{A \times C}{c}\right) \qquad \text{I}$$

But $\frac{C}{c} = \frac{1871206920000000}{1577916450000} = \frac{420024000}{35419}$ dividing by the common factor 44550000

∴ The space described by any planet is from I

$$A \times 11859 - A \left\{\frac{c \times 11859 - C}{c}\right\}$$

$$= A \times 11859 - A \left\{\frac{35419 \times 11859 - 420024000}{35419}\right\}$$

But the numerator within the brackets is 9921

∴ Space described $= A \times 11859 - \frac{A}{-} \times$

as stated. This distance divided by the individual orbital lengths, we have the integral number of revolutions made by each planet, rejecting which we have the fractional part of a revolution which gives the position of the planet.

Verses 8, 9. The orbit of the planet itself is no doubt the orbit of the Mandoccha (apogee with respect to the Sun and the Moon and aphelion with respect to the other Star-planets ie Mars, Mercury, Jupiter, Venus and Saturn) and of the node (point of intersection of the orbits of the Star-planets with the ecliptic); but while Computing the positions of these Mandocchas and the Nodes, as per the method indicated above ie according to the method of

Kakshādhyāya, their orbits are taken to differ from those of the planets. (because slow-moving points will have longer orbits as per the assumption made namely that the circumference of the universe divided by the number of the sidereal revolutions gives the length of the orbit). Similarly the orbit of the Sun itself will be the orbit of Mercury and Venus, and the orbits of their S'ighrocchas are their real orbits wherein Mercury and Venus are taken to move with the velocity of the Sun.

Comm. The prescription of the computation of a planetary position as per the method of Kakshādhyāya, has brought in an awkward situation. Let us consider the Computation of the positions of Mercury and Venus. These two planets oscillate about the mean position of the Sun, because their orbits happen to lie within the earth's orbit. Hence their mean sidereal periods coincide with that of the Sun, which means that their numbers of sidereal revolutions coincide with the number of sidereal revolutions of the Sun. Hence the Kakshādhyāya method of Computing a planetary position brings in the idea that the orbits of Mercury and Venus coincide with the orbit of the Sun. Bhāskara perceived the awkwardness of this situation and says therefore, the above coincidence of the orbits must not be taken to be a reality but is intended only for the sake of computation. The actual planets Mercury and Venus in fact revolve says Bhaskara in the orbits of their Sīghrocchas, with the velocity of the Sun-Even this supposition that the actual planets move with the velocity of the Sun sounds odd, but this will be clarified later in the spashta-adhikāra, wherein we propose to explain the peculiar concept of S'ighroccha at length.

Here ends the Grahānayanādhyāya according to the Kakshādhyāya method.

THE ADHYĀYA KNOWN AS PRATYABDA S'UDDHI IN MADHYĀDHIKĀRA

Verse 1. The number of years which have elapsed from the beginning of the Kalpa, respectively multiplied by 2, 4 and 3 and divided by 8, gives what is called Dinādya in days, ghatis and Vighatis respectively. If this be added to the number of years and divided by seven, the remainder gives the Abdapa or the lord of the year, (under whose name the week-day of the commencement of the year stands).

Comm. One mean solar year consists of 365 days, 15 ghatis, 30 palas, and $22\frac{1}{2}$ Vipalas where the units are all mean solar and one mean solar day is equal to 60 mean solar ghatis, one mean solar ghati is equal to 60 mean solar palas, one mean solar pala is equal to 60 mean solar Vipalas and so on in sexagesimal sub-division. The fraction of the day over and above 365 days, namely 0–15–30–22–30 multiplied by 8 gives 2 days, 4 ghatis and 3 palas so that by the rule of three i.e. 'If in 8 mean solar years, the fraction accrues to 2 days, 4 ghatis and 3 palas, what will it accrue to in x elapsed mean solar years from the beginning of the Kalpa? we have the answer $\frac{x \times 2}{8}$ days, $\frac{x \times 4}{8}$ ghatis and $\frac{x \times}{8}$ palas. If this is added to the number of the elapsed years, and the result divided by seven, the remainder gives the week-day of the commencement of the concerned year, because the remainder got by dividing 365 by 7 is one, and the week day advances at the rate of one per year. Also the Kalpa began on Sun-day.

Verse 2. Alternate method.

Half the number of elapsed years added to $\frac{1}{80}$ of itself, then divided by 60 and added to $\frac{1}{4}$ of the elapsed years, gives the Dinādya.

Comm. Since the Dinādya per year is 0–15–30–22–30, in x elapsed years it accrues to x × 0–15–30–22–30 $= \frac{x \times 15}{60}$ days $+ \frac{x \times 30}{60}$ ghatis $+ x \times 22\frac{1}{2}$ palas $= \frac{x}{4}$ days$+$ $\frac{x}{2}$g $+ \frac{x \times 45}{2 \times 60}$ palas $= \frac{x}{4}$d $+ \frac{x}{2}$g $+ \frac{3x}{8} \times \frac{1}{60}$g $= \frac{x}{4}$d $+$ $\frac{x}{2}(1+\frac{1}{80})$g $= \frac{x}{4}$d $+ \frac{x}{2}\frac{(1+\frac{1}{80})}{60}$d which is the given formula.

Verse 2. Alternative method.

The number of elapsed years divided by respectively 4, 120, and 9600 and the Sum taken gives the Dinādya.

Comm. Let x be the number of elapsed years. The Dinādya as before is x×0–15–30–22–30 $= \frac{x}{4}$d $+ \frac{x \times 30}{60 \times 60}$d $+ \frac{x \times 45}{2 \times 60} \times \frac{d}{60 \times 60} = \left(\frac{x}{4} + \frac{x}{120} + \frac{x}{9600}\right)$d as given.

Verse 3. To obtain what is known as Kshayāhādya. The number of elapsed Kshayāhās from the beginning of Kalpa upto the commencement of the year, is obtained as follows. Let x be the number of elapsed years; then $x - \frac{x(1+\frac{1}{60}) + 30x}{160}$ = Kshayāhās.

Comm. The number of Kshayāhās in a Kalpa of 4320000000 solar years is 25082550000 so that per year their number is 5–48–22–7–30. In this 0–48–22–7–30 is said to be Kshayāhādya per year = 1 – (0–11–37–52–30) putting the quantity within the brackets into a fraction, $52\frac{1}{2}$ vipalas $= \frac{105}{2} \times \frac{1}{60} = \frac{7}{8}$ palas; $(37+\frac{7}{8})$ palas $= \frac{101}{8} \times \frac{1}{60}$ ghatis $= \frac{101}{160}$ ghatis; $11\frac{101}{160}$ ghatis $= \frac{1861}{160 \times 60}$ days

160 of $\frac{1861}{60}$ days $= \frac{1}{160}$ of 31 days, one ghati. Hence

per year the Kshayāhādya is $1 - \frac{1}{160}$ of $\overset{d}{(31}\overset{g}{-1)}$ so that

for x years it would be $\left\{x - \frac{x}{160}\left(\overset{d}{31}\overset{g}{-1}\right)\right\}$ d

$$ 160$$

$$= x - \frac{1}{160}\left\{30x + x\left(1 + \tfrac{1}{60}\right)\right\}$$ which is the formula given.

4. Alternative method.

The Dinādya obtained before multiplied by three, is to be diminished by $\frac{1}{400}$th of the number of years; the result increased by $\frac{1}{30}$th of the number of years gives the number of Kshayāhās.

Comm. The Dinādya pertaining to one year is 0-15-30-22-30 and the Kshayāhādya is 0-48-22-7-30. Multiply the former by 3 and subtract from the latter; we have $0\text{-}1\text{-}51 = 0\text{-}1\text{-}\frac{51}{60} = 0\text{-}1\text{-}\frac{17}{20} = 0\text{-}\frac{37}{20} = \frac{37}{20} \times \frac{1}{60} = \frac{37}{1200}$ day Hence $K - 3D = \frac{37}{1200}$ day where K = Kshayāhādya and D = Dinādya $\therefore K = 3D + \frac{37}{1200}$ hence per x years it

$$= 3D \times x + \frac{x}{30} - \frac{x}{400}$$ which is the formula given.

4 contd. Alternative method.

Comm. The Kshayāhādya for an year is 0-48-22-7-30 48 ghatis $= (1 - \frac{1}{5})$ day; hence for x years $x(1 - \frac{1}{5})$.

The remaining fraction $= 0\text{-}0\text{-}22\text{-}7\tfrac{1}{2} = 0\text{-}0\text{-}(22+\tfrac{15}{2}\times\tfrac{1}{60}$

$= 0\text{-}0\text{-}22\tfrac{1}{8} = 0 - \dfrac{177}{8} \times \dfrac{1}{60} = 0 - \dfrac{59}{160} = \dfrac{60-1}{160 \times 6}$

$\text{day} = \dfrac{1}{160} - \dfrac{1}{60 \times 160} = \dfrac{1}{160}\left(1 - \dfrac{1}{60}\right)$; hence for :

years $\dfrac{x}{160}\left(1 - \dfrac{1}{60}\right)$

Adding the two we have the required formula.

Verse 5. To obtain the elapsed number of Adhikamāsās and what is called S'uddhi.

The sum of the Dinādya, Kshayāhādya and ten times the number oi elapsed years divided by 30, gives the number of the elapsed Adhikamāsas; the remainder is known as S'uddhi if diminished by the fraction of the Kshayāhas.

Comm. The number of mean solar days in an year is 365-15-30-22-30; the Kshayāhās in an year are 5-48-22-7-30; adding the two we have the number of tithis in an year equal to 371–3–52–30. The number of solar days being 360, the number of tithis which constitute the Adhimāsas is equal to 11–3–52–30. The sum of the Dinādya and Kshayāhādya in an year = 0–15–30–22–30 + 0–48–22–7–30 = 1–3–52–30. Hence the above number of tithis which constitute the Adhimāsas namely 11-3-52-30 = 10 + Sum of Dinādya and Kshayāhādya pertaining to an year. Hence for x years, the Adhimāsa days are equal to x×10+Sum of Dinādya and Kshāhādya for x years. These Adhimāsa days divided by 30 give the Adhimāsas, and the remainder is called S'uddhi, so called because while finding the Ahargaṇa from the beginning of a solar year, this remainder has to be subtracted. Why the fraction of Kshayahas, which is there in this S'uddhi is prescribed to be subtracted, will be explained in another context.

Verse 6. An alternative method to find the Adhimāsas. The number of years divided separately by 32 and 30, the sum increased by the number of years multiplied by eleven and the result divided by 30 gives the number of elapsed Adhimāsas. The remainder diminished by the fraction of Kshayāhās as mentioned before, is the S'uddhi.

Comm. Every year, the number of Tithis that constitute the Adhimāsās accruing in the course of years, is 11-3-52-30. The fractional part of this namely 0-3-52-30

$$= 0\text{-}3\text{-}52\tfrac{1}{2} = 0 - 3 + \frac{105}{2\times 60} = 0 - 3\tfrac{7}{8} = 0 - \frac{31}{8}$$

$$= \frac{31}{8} \times \frac{1}{60} = \frac{31}{480} = \frac{16+15}{480} = \frac{1}{30} + \frac{1}{32}$$ converted

into days. Hence for x years $\frac{x}{30} + \frac{x}{32}$. Adding the

number of days 11x, we have $11x + \frac{x}{30} + \frac{x}{32}$ as stated.

Dividing these days by 30, we have the number of elapsed Adhimāsas and the remainder is S'uddhi, if the fraction of the Kshayāhās is subtracted therefrom as indicated.

Verse 7. Method of finding the lord of the year without a knowledge of Dinādya.

The week-day at the Commencement of the Solar year, which pertains to the lord of the year, is the remainder got by dividing by seven the S'uddhi which is itself diminished by the remainders got by dividing by seven separately firstly double the excess of the elapsed years over the elapsed Adhikamāsās and secondly the elapsed Kshayāhās. This may be put in symbols as $R_7 \{ S - R_7 (2y - A) - R_7 . K. \} = R_7 \{ S' - R_7 (2y - 2A + K) \}$ where R_7 signifies the remainder got by dividing by seven the quantity which follows, S signifies S'uddhi, y means the elapsed Solar years A the elapsed Adhikamāsās and K the elapsed Kshayāhās.

Comm. The lord of the year is the lord of the week-day at the Commencement of the solar year. To get this week-day we have if the number of Tithis elapsed upto the beginning of the luni-solar year be T, the Suddhi S′, the Kshayāhās upto the Commencement of the solar year be K, then since the number of civil days is equal to T + K, R_7 (civil days) = R_7 (T + S — K). But T = 360y + 30A where y is the number of elapsed solar years and A the number of elapsed Adhikamāsās. ∴ R_7 (T) = R_7 (360y + 30A) = R_7 (3y + 2A) since the remainder got by dividing 360 and 30 by seven are respectively 3 and 2. Hence R_7 (civil days = Ahargaṇa) = R_7 (3y + 2A) + R_7S' − R_7K. The number of Kshayāhās K = y (5-48-22-7-30) = 5y + y (0-48-22-7-30). But y (0-48-22-7-30) = K where K is the previously calculated Kshayāhādya. Hence R_7K = $R_7 5y$ + R_7K. Substituting this in the above R_7 (civil days) = R_7 (3y + 2A) + R_7S' − R_7 (5y + K) = R_7S' − R_7 (5y — 3y) + R_7 (2A) − R_7K = R_7 (S′ — R_7 (2y — 2A) — R_7K] which is the given formula = R_7 (S′ — R_7 (2y — 2A + K)].

Verse 8. To obtain the fractional part of the elapsed Kshayāhās even without a knowledge of the number of Kshayāhās.

The excess of the ghatis of the Adhimāsa Sesha ie the remainder got by dividing the sum of the Dinādya, Kshayāhādya and ten times the number of elapsed years by 30 as indicated in verse 5, over the ghatis of the Dinādya, obtained under verse 1 gives the fractional part of the Kshayāhās.

Comm. The ghatis or the fractional part of the masa Sesha is the sum of the fractional parts of Dinādha and the Kshayāhādya so that the excess of the ghatis of the Adhimāsa Sesha over the ghatis or the fractional part of the Dinādya gives the ghatis or the fractional part of the Kshayāhās.

Verse 9. The planetary positions at the end of the elapsed solar year.

The number of the elapsed solar years multiplied by the number of sidereal revolutions of the respective planets in a Kalpa and divided by the number of Solar years in a Kalpa gives the planetary positions at the end of the last Solar year.

Comm. This is a simple rule of three. The planetary positions so got, leaving out the integral numbers of revolutions made which are not required, are called the Dhruvakas of the respective planets for the ensuing solar year. With respect to the apogee of the Sun and the aphelia and nodes of the star—planets these Dhruvakas themselves give their positions for the whole ensuing year, as their motion is very very slow.

Verse 10. An alternative method of obtaining the Dhruvaka of the Moon.

The Adhimāsa Sesha multiplied by 12 gives the position of the Moon at the Commencement of the Solar year.

Comm. Since the position of the Sun is at the Zero-point of the Zodiac at the Commencement of the solar year, and since the Adhimāsa Sesha is the difference of the solar and luni-Solar systems of reckoning, or what is the same, the arc gained by the Moon over the Sun, which is no other than the elongation of the Moon measured in Tithis each tithi being of 12° gain of elongation, so the Adhimāsa Sesha at the Commencement of the solar year in Tithis multiplied by 12 gives the longitude of the Moon at that Commencement. (Note - Bhaskara waxes into poetic eloquence in the second half of the verse, having given the procedure in the first half).

Verse 11. Procedure prescribed in the event of computing planetary positions from the beginning of the Kaliyuga for the sake of convenience.

The Dinādya may be also obtained from the beginning of the Kali, which begins with Friday. The Dhruvas calculated for the commencement of the solar year are to be added to the planetary positions at the beginning of the Kali, in the event of computing the Ahargaṇa and thereby the planetary positions from the beginning of the Kaliyuga for the sake of convenience.

Comm. The Dinādya at the commencement of Kali is Zero, since the number of civil days during the length of time equal to a Kaliyuga is integral and equal to 157791645 according to Brahmagupta and Bhaskara who follows him. The Suryasiddhānta, it may be noted here, gives the number of days in a Kaliyuga as not integral but equal to 157791782.8. Hence the Dinādya computed from the beginning of the Kali is to be increased by .8 to obtain its value according to Suryasiddhānta. The planetary positions at the commencement of Kali were given by Bhaskara already.

Verse 12. The number of what are called Kshepadinās to obtain the Ahargaṇa.

Hereafter Bhaskara is going to obtain the planetary positions for any day during the current solar year having obtained the Dhruvakas or the planetary positions for the beginning of the Solar year. In that behalf the Ahargaṇa or the collection of days which have elapsed from the commencement of the Solar year is to be found. This Ahargaṇa is obtained by subtracting the number of Kshayāhās from the number of tithis that have elapsed. In finding these Kshayāhās, we have to take note that there is a little remnant of Kshayāhās at the beginning of the Solar year which is also to be taken into account while computing the number of Kshayāhās during the course of the year. In other words, the number of Kshayāhās that are going to be computed during the course of the year, for the elapsed part of the year will be in default of the actual number if we ignore the accrued fraction of Kshayāhās at the commencement of the Solar year. To make amends for that default we have to add some number to the numerator of the improper fraction which is going to give us the number of the elapsed Kshayāhās during the course of the year. The formula that is going to be used to obtain the number of Kshayāhās during the elapsed tithis is $\frac{x}{64}\left(1+\frac{1}{702}\right)$. This formula arises out of the fact that there are 55739 Kshayāhas during 3562220 tithis, so that for 64 tithis the number of Kshayāhās is equal to $\frac{64 \times 55739}{3562220} = \frac{3567296}{3562220} = 1 + \frac{5076}{3562220} = 1 + \frac{1}{\frac{3562220}{5076}} = 1 + \frac{1}{702}$. Let the fraction of Kshayāha at the commencement of the solar year, to be taken into account be x ghatis i.e. $\frac{x}{60}$ of a tithis (for Kshayāhās are are computed out of tithis) i.e. $\frac{x \text{ tithis}}{60}$. Let y be the number of tithis elapsed after the commencement of the solar year. Then to compute the Kshayāhās that ensue after the commencement of the solar year upto the day concerned during the course of the year, the formula to be used is $\frac{y}{64}\left(1+\frac{1}{702}\right)$. To this we have to add $\frac{x \text{ tithis}}{60}$ as the balance of Kshāyāha at the commencement of the solar year to be taken into acccount

$$\frac{x \text{ tithis}}{60} = \frac{x \times 64}{64 \times 60} = \frac{x \times \frac{64}{60}}{60}$$

Now, in computing the number of tithis which have elapsed after the commencement of the Solar year, we subtract S'uddhi from the number of tithis that have elapsed after the beginning of the luni-Solar year. But in this S'uddhi we have subtracted the fractional part of the Kshayāhas for a different purpose so that in subtracting the S'uddhi, we have increased the tithis by the fractional part of the Kshayāhās. This increase therefore should be nullified, which means that the quantity to be added is

$$\frac{x \times \frac{64}{60}}{64} - \frac{x}{60} = \frac{x \times \frac{63}{60}}{64} = \frac{21}{20}\frac{x}{64} = \frac{x\,(1+\frac{1}{20})}{64}$$

Hence the Kshepadinas or the tithis to be added to the number of tithis which have elapsed from the commencement of the Solar year, are $x\,(1+\frac{1}{20})$. This means that the ghatis x which form the fractional part of the Kshayāhas are to be increased by one twentieth part of themselves and are to be viewed as tithis and not ghatis as mentioned.

Verse 12 (contd.) and *Verse* 13. To find the Ahargaṇa, ie the collection of days which have elapsed from the commencement of the solar year.

The number of tithis which have elapsed from the commencement of the luni-Solar year diminished by the S'uddhi, increased by $\frac{1}{702}$th part of the result, and then increased by the Kshepa-tithis aforesaid, and the result divided by 64, gives the number of Kshayāhās, which have elapsed from the beginning of the Solar year. These are to be subtracted from the tithis which have elapsed from the beginning of the Solar year to give the Ahargaṇa.

Fig. 3

Comm. (Refer Fig. 3). Let T_1 be point indicating the commencement of luni-Solar year ; let R_1 be the point indicating the next Sun-rise. Let S be the beginning point of the Solar year, R_2 the preceding Sun-rise and T_2 the ending moment of the preceding tithi ; Let R_3 be the next Sun-rise, T_3 the ending moment of the tithi preceding the Sun-rise R_4 upto which point the Ahargaṇa has to be found from the commencement of the Solar year. Thus the Ahargaṇa to be found is $SR_4 = ST_3 + T_3 R_4$. Here $T_3 R_4$ is the Kshayāghatis, at the Sun-rise concerned. SR_3 is a fraction of a day that is to be there in the Ahargaṇa SR_4 we are seeking. While subtracting the S'uddhi $T_1 S$ diminished by the Kshayāghatis $T_2 R_3$ from the number of elapsed integral number of tithis from the commencement of the luni-Solar year namely $T_1 T_3$ we have $T_1 T_3 - (T_1 S - T_2 R_3) = T_1 T_3 - (T_1 T_2 - SR_3) = T_2 T_3 + SR_3$. Thus instead of $SR_3 + R_3 R_4$ we have by the above procedure $SR_3 + T_2 T_3$. Though both $T_2 T_3$ and $R_3 R_4$ are integral numbers and should be the same if the interval is small, they may differ by an integral number, if the interval happens to be long. Bhaskara says that this difference of an integral number will be rectified by the Kshepadinas found under verse No. 12, for, from these Kshepadinas, the Kshayāhas are found and subtracted from the tithis. It will be noted that the difference of an integral number between $T_2 T_3$ and $R_3 R_4$ is no other than Kshayāhas.

Or, this may be seen in an other way. We are to find $SR_4 = T_1 R_4 - T_1 S = T_1 T_3 + T_3 R_4 - T_1 S = T_1 T_3 - (T_1 S - T_3 R_4)$ = No. of elapsed tithis — (S'uddhi — Kshayaghatis at the day concerned). But instead of the Kshayaghatis at the day concerned Bhaskara prescribes the subtraction of the Kshayāghatis at the end of the Solar year ie instead of subtracting $T_3 R_4$ it is prescribed to subtract $T_2 R_3$. This difference, Bhaskara says is made up by taking into account the Kshepadinas pertaining to the Kshayāhas.

Verse 14. In case the Ahargaṇa is required for a day preceeding the commencement of the Solar year, then the elapsed tithis are less than the Tithis of the Suddhi; so subtraction is not feasible. In this case, it is prescribed to take the elapsed tithis from the previous Chaitra and the S'uddhi of the previous year. Also in this case the Dhruvakas pertain to the commencement of the previous Solar year.

Comm. Easy.

Verse 15. To obtain the position of the Sun.

The number of the days in the Ahargaṇa is to be diminished by $\frac{1}{60}$ part of itself to obtain the number of degrees, and fractions thereof; then the Ahargaṇa multiplied by three and divided by 22 gives the minutes and fractions thereof. Adding the two results we get the position of the Sun.

Comm. The mean daily motion of the Sun is 0–59–8–10–21. Here $59' = 1° - 1' = (1 - \frac{1}{60})°$; for x days $x\,(1 - \frac{1}{60})° = x° - \frac{x°}{60}$ as mentioned. The remaining part namely $0\text{–}0\text{–}8\text{–}10\text{–}21 = \frac{9807'}{72000}$; Converting this into a continued fraction it is equal to $\frac{1}{7+}\,\frac{1}{2+}\,\frac{1}{1+}\,\frac{1}{12+}\cdots$ of which a good convergent is $\frac{3'}{22}$. Hence for x days $\frac{3x}{22}$ as stated in the verse.

Verse 16. To obtain the position of the Moon.

The number of elapsed integral tithis multiplied by 12 and added to the Sun's position in degrees, gives the Moon's position in degrees at the ending moment of the tithi preceeding the day at the Sun-rise of which the planetary positions are sought. To find the position at the Sun-rise required, ten times the Kshaya-dina-Sesha increased by $\frac{1}{7}$th of itself gives the number of minutes to be added to the position got above.

Comm. The first part is clear, because for every tithi, there will be an increase of elongation of 12°. To get the fractional part of the tithi in between the ending moment of the tithi and the subsequent Sun-rise, the interval which is Sāvana is expressed in the unit of civil day has to be converted into luni-Solar units. The rule of three is 'If for 63 Sāvana days there are 64 tithis, what will it be for the above interval ?' Here the approximate ratio of $\frac{64}{63}$ is used because the Ahargaṇa which is Laghu is small ie is less than 365. The Kshaya-dina-Sesha which was got under verses 12, 13, has a divisor of 64, and is of the form $\frac{y}{64}$.

Hence the quantity to be added is $\frac{y \times 64}{64 \times 63} = \frac{y}{63}$ of a tithi $= \frac{y}{63} \times 12 \times 60$ minutes of arc $= \frac{8}{7} \times 10\,y = 10\,y\,(1 + \frac{1}{7})$ minutes as given in the verse. This has to be added to 12t°, got before where t is the number of elapsed tithis.

Verse 17. Computation of Mars.

The mean daily motion of Mars is 0–31–26–28–7=0–30 + 0–0–90 minus 0–0–3–31–53. If x be the number of days $x \times 0\text{–}30' = \frac{x^\circ}{2}$; $x \times 0\text{–}0\text{–}90'' = \frac{x}{2} \times 3'$; thus the rule prescribed is "Half of the number of days gives the degrees; half the number of days multiplied by 3 gives the number of minutes" from this we have to subtract x × (0–0–3–31–53). The quantity within the brackets is approximately $\frac{1}{17}$ of a minute for $\frac{1}{17}' = 0\text{–}0\text{–}3''\text{–}31'''\text{–}46''''$. Since the Ahargaṇa is small the precision required is there. Thus the rule is $\frac{x^\circ}{2} + \frac{x}{2} \times 3' + \frac{x'}{17} +$ the Dhruvaka at the beginning of the Solar year.

Verse 18. Computation of the S'ighroocha of Mercury.

The mean daily motion of the Budha-S'ighra is 4°–5′–32″–18‴–28⁗.

Rule. If the Ahargaṇa be x days then the S'ighra will be $4x^\circ + \frac{4x \times 3'}{130}$ + Dhruvaka. *Proof.* For x days, the mean motion is $x \times 4^\circ + x \times (0\text{-}5'\text{-}32''\text{-}18'''\text{-}28'''')$. The second part is taken to be $4x \times \frac{3^\circ}{130}$ ie $x \times \frac{12^\circ}{130} = x \times \frac{6}{65}^\circ$ for $\frac{6}{65}$ of a degree $= 0\text{-}5'\text{-}32''\text{-}18\text{-}'''28''''$. The fraction $\frac{6}{65}$ can be seen to be a convergent of the remainder for $5'\text{-}32''\text{-}18'''\text{-}28'''' = \frac{299077^\circ}{3240000} = \frac{1}{10+}\frac{1}{1+}\frac{1}{4+}\frac{1}{1+}\frac{1}{9968}$ of which the penultimate convergent is $\frac{6}{65}$.

Verse 19. The Ahargaṇa divided by 12 and by 71 gives respectively the positive degrees and negative minutes to be added to the Dhruvaka of Jupiter.

Comm. If x be the Ahargaṇa $\frac{x^\circ}{12} - \frac{\bar{x}}{71}$ + Dhruvaka = Guru.

The mean daily motion of Jupiter is 0-4-59-9.9=0-5′ minus 0-0-0-50-51. Evidently $0\text{-}5' \times x = \frac{x^\circ}{12}$. The remainder $0\text{-}0\text{-}0\text{-}50\text{-}51 = \frac{61'}{4320}$ of which the continued fraction is $\frac{1}{70+}\frac{1}{1+}\frac{1}{4}$. A very approximate convergent is $\frac{1}{71}$ as given.

Verse 19 contd. To obtain the position of the S'ighra of venus.

The Ahargaṇa multiplied by 10 and divided by 6 and 155 respectively gives degrees positive and negative to be added to the Dhruvaka of Venus to give his position.

Comm. The formula is $\frac{10\,x}{6} - \frac{10\,x}{155}$. The mean daily motion of the S'ighra of Venus is $1^\circ\text{-}36'\text{-}7''\text{-}44'''\text{-}35'''' =$

1°–$40'$ minus 0–3′052″–15‴ approximately. For x days x $\times$ $1\frac{2^\circ}{3} = \frac{5}{3}x^\circ = \frac{10\,x^\circ}{6}$ which gives the first part. Also 3′–52″–15‴ $= \frac{929^\circ}{14400} = \frac{1}{15+}\,\frac{1}{1+}\,\frac{1}{1+}\,\frac{1}{464}$ of which a proximate convergent is $\frac{2}{31} = \frac{10}{155}$ ie for x days $\frac{10x^\circ}{155}$. Hence the position is given by $\frac{10\,x^\circ}{6} - \frac{10\,x^\circ}{155}$ + Dhruvaka.

Verses 20. The position of Saturn is given by $\frac{2x'}{5}$ + $\frac{2x''}{5}$ + Dhruvaka = Saturn's position.

Comm. The mean daily motion of Saturn is 0–2–0–22–51.

For x days 2x′ + x $\times$ 0″–22‴–51⁗. The latter part is $\frac{137''}{360}$ approximately $= \frac{1}{2+}\,\frac{1}{1+}\,\frac{1}{1+}\,\frac{1}{1+}\,\frac{1}{2}$ of which a near covergent $\frac{2}{5}$ so that for x days, we have 2x′ + $\frac{2x''}{5}$ + Dhruvaka position of Saturn.

Verses 20 contd. To find the position of the apogee of Moon.

The Ahargaṇa divided successively by 10 and 88 and added gives the degrees to be added to the Dhruvaka of the Apogee of the Moon to obtain its position.

Comm. The mean daily motion of the apogee of the Moon is 0–6–40–53–56. At the rate of 6′ per day, in x days the number of degrees covered is $\frac{x}{10}$; the remainder

$$\frac{4601^\circ}{} = \frac{1}{88+}\,\frac{1}{41} \text{ of }$$

approximate convergent is $\frac{1}{88}$. Hence the position is given by $\frac{\Delta}{10} + \frac{x^\circ}{88}$ + Dhruvaka as given by the verse.

Verse 21. To obtain the position of the lunar Node

The Ahargaṇa multiplied by 30 and divided by 566 gives the number of degrees to be added to the Dhruvaka to give the position of Rahu.

Comm. If the number of the sidereal revolutions of Rahu in a Kalpa multiplied by twelve divide the number of days in a Kalpa we have very approximately 566, which means that Rahu traverses a Rasi very nearly in 566 days. Hence dividing the Ahargaṇa by 566, and multiplying by 30, we have the degrees covered, which being added to the Dhruvaka gives the position of Rahu.

22, 23, 24. Alternative method of obtaining the planetary positions.

The Ahargaṇa multiplied by 100000, and divided successively by 101461, 151787, 190833, 24436, 1203400, 62416, 2990000, 898000, 1886800 gives the respective positions of theplanets beginning from the Sun and those of the apogee and Node of the Moon; in the case of the Moon, however, the result is to be multiplied by 20. The results in degrees added to the Dhruvakas give their positions.

Comm. The degrees covered by the planets etc in D days are $\frac{R \times 360 \times D^\circ}{M}$ where R is the number of sidereal revolutions in a Kalpa, M the number of mean solar days in a Kalpa, and D the Ahargaṇa. Now

$$\frac{R \times 360 \times D}{M} = \frac{R \times 360 \times D \times 100000}{M \times 100000} = \frac{D \times 100000}{\frac{M \times 100000}{R \times 360}}.$$

Then $\frac{M \times 100000}{R \times 360}$ is found for every planet. In the case of the Moon, however a multiplier 20 also is used in addition to 100000, because he has such a quicker motion.

Verses 25, 26. To obtain the mean daily motion of the planets.

The number of minutes of arc moved by a planet per day gives the mean daily motion of that planet. Though, however, the spatial velocity of each planet is the same per day, in angle, the velocities differ, (on account of the varied distances) and so we perceive slowness or fastness in the movement of the planets.

Comm. Easy.

The assumption of equal daily spatial velocities for all the planets has been explained before. The daily angular velocity of a planet in minutes of arc is $\frac{R \times 360 \times 60'}{M}$ where R is the number of its sidereal revolutions and M the number of mean solar days in a Kalpa.

Verse 27. The reason for unequal angular velocity of the planets.

Since the planetary orbits are all construed as comprising of 360° alone, a minute of arc of a smaller orbit has a smaller spatial distance which will be covered more rapidly, the velocity being constant, whereas of a longer orbit, a minute of arc means a longer distance which will be covered in a longer time at the same spatial velocity, which means that the planet appears to be slow in motion. Thus the Moon, the Mercury, Venus, Sun, Mars, Jupiter, Saturn being placed at longer distances from the earth in ascending order their orbits are longer in ascending order so that they are slower in angular motion in that order.

Here ends the section called Pratyabda S'uddhi in the chapter Madhyādhikāra.

THE SECTION KNOWN AS ADHIMĀSĀDI-NIRNAYA IN Ch. 1

Verse 1. A special feature of the computation of Ahargaṇa. If the Ahargaṇa is to be increased or decreased by unity to adjust it with the week-day, the tithis also are to be increased or decreased by unity to be adjusted with the week-day. Then in that context of increasing or decreasing the Ahargaṇa by a day the Adhimāsa-Sesha is to be increased or decreased by the number of Adhimāsās of a Kalpa and the Avama-Sesha is to be increased or decreased by the Avamās or Kshayāhās of a Kalpa.

Comm. After having computed the Ahargaṇa as per the directions given under verses 1–3 of the section of Grahānayana, we have to test its correctness on the basis of the week-day taking it that the Kalpa began with Sunday. In other words, since the number of the elapsed week-days accord with the number of Sun rises, the week-day on the day on which the Ahargaṇa is computed should accord with the Ahargaṇa ie dividing the Ahargaṇa by seven, the remainder must give us the week-day of the day in question. But it so happens that we may have to add or subtract one from the Ahargaṇa arrived at to adjust the Ahargaṇa with the week-day. Why does this happen ?

The reason is as follows. When we find the Ahargaṇa for a particular number of elapsed tithis, we are unwittingly taking the number of elapsed true tithis in the place of the average tithis. The fact that we are given in the table of constants, the number of mean units of time in a Kalpa, for example mean solar days, mean tithis etc, and the fact that computation proceeds only on the basis of these mean units, means that we have taken into account only the number of mean tithis elapsed. Hence this is to be corrected by us. If we are computing the Al for, say, the pratipat of phālguna, we automatically take that the elapsed true tithis from the beginning of the luni-Solar year is $11 \times 30 = 330$ tithis. But it may so happen that had we taken only the number of mean tithis elapsed, either 229 tithis or 331 tithis might have elapsed and not 330 as construed which therefore works an error amounting to unity in the Ahargaṇa arrived at. At the same time the error does not exceed unity, for, during the course of every lunation, the longer tithis are almost compensated by tithis of shorter duration in the same lunation. The variance in the length of a tithi being wrought by the variable motion of the Moon, which again depends on the distance of the Moon from his apogee, is rounded off in every lunation, as the Moon completes a circle with respect to the apogee in what is called an anomalistic month.

Thus it is that the Ahargaṇa is to be rectified on the basis of the week day. When this is done and the Ahargaṇa happens to be increased or decreased by unity, automatically it goes without saying, that the tithis are also increased or decreased by one. In this context there is a still more deeper significance as detailed below. The Adhimāsās are computed from the following formula viz. $\frac{A \times a}{S} = I + \frac{F}{S} \times 30$ (1) where A = Ahargaṇa, a = adhimasās in a Kalpa, S = solar days in a Kalpa, I = Integral quotient obtained by division and $\frac{F}{S} \times 30$ the Adhimasa-Sesha-tithis. If now A is to be increased or decreased by 1 $\frac{(A \pm 1)\,a}{S} = \frac{Aa}{S} \pm \frac{a}{s} = I + \frac{F}{S} \times 30 \pm a/S$ from (1) $= I + \frac{F \times 30 \pm a}{S}$; hence the adhimāsa-sesha namely $F \times 30$ is increased or decreased by a ie the number or adhimasās.

Similarly the Avama-Sesha or the Kshayāha-Sesha is to be increased or decreased by the Kshayāhas in a K...

The particular mention of the increase or decrease in the Adhimāsa-Sesha or the Avama-Sesha is necessitated in the context of computing the positions of the Sun or Moon, given the Adhimāsa Sesha and Avama-Sesha as mentioned under verses 6, 7 in the section of grahānayanā.

Verse 2. Pertaining to the smaller Ahargaṇa computed from the beginning of the current Solar year, called Laghu-Ahargaṇa.

In the case of computing the Ahargaṇa from the beginning of the Solar year also, when the Ahargaṇa is to be increased or decreased by unity, the tithis are to be increased or decreased by unity. The Avama-Sesha here is to be increased or decreased not by the Kshayāhās of the Kalpa but only by unity because we have used the formula $\frac{1}{64}$ of tithis to get the Kshayahās. If, an Adhikamāsa happens to occur during the course of the current year, the tithis, 30 in number of this Adhikamāsa must be also taken into account to obtain the Ahargaṇa.

Comm. Easy.

Verse 3, 4. The Ahargaṇa is to be computed (larger Ahargaṇa) after taking into account an Adhimāsa which has conspicuously occured but which is not obtained by computation or by rejecting an Adhikamāsa which has not occured but which is obtained by calculation. The Adhimāsa-Sesha is to be increased or decreased by the Adhimāsas of the Kalpa; the elapsed months from the beginning of the luni-Solar year are to be increased or decreased by unity and then the positions of the Sun and the Moon are to be computed from such an Adhimāsa-Sesha and such an Avama-Sesha.

Comm. Already explained.

Verse 5. A point to be noted with respect to the S'uddhi.

In the case of obtaining the S'uddhi, if an Adhikamīsa, which did not actually occur, is obtained by calculation, then the S'uddhi is to be increased by 30, so that the Ahargaṇa is not affected by the un-occuring Adhikamāsa.

Comm. The computation of the Adhikamāsas or intercalary months proceeds under the consideration of mean lengths. So, it is likely that an Adhikamīsa may occur un-warranted by calculation or may not occur in spite of its being shown by calculation. Further, an Adhikamāsa may be delayed in occurence by the fact that though the luni-Solar reckoning has gained over the solar by one mean lunation, the lunation at that point may still contain a Samkrānti, the preceding particular lunar month being smaller in length than the mean. Thus the convention made with respect to the occurence of an adhikamāsa, namely that the lunation which does not carry a Samkrānti is to be construed as an adhikamāsa, may also delay the occurence of the Adhikamāsa, though shown in calculation. Similarly an Adhikamāsa may be preponed though not warranted by computation by the same logic.

Verse 6. The criteria of an Adhikamāsa and a Kshayamāsa.

A lunation which does not carry a Samkrānti is an Adhikamāsa; whereas a lunation which carries two Samkrāntis is to be taken as a Kshayamāsa. The Kshayamāsa, occurs only in the course of the three lunar months named Kārtica, Mārgasīrsha and pausha and not during any other lunar month; when a Kshayamāsa occurs, then during the course of that year there will be two Adhikamāsās occuring on either side of the Kshaya-

Comm. The institution of intercalation has been explained to some extent under verse 10 of the Bhagaṇādh-
We shall see some more particulars of intercalation.

1. Bhaskara has given that 1593300000 Adhikamāsās occur in a Kalpa of 4320000000 Solar years, which means that 15933 Adhikamāsās occur in 43200 Solar years ie. 5311 Adhikamāsas in 14400 solar months. Converting $\frac{14400}{}$ into a continued fraction, we have $2 + \frac{1}{1+} \frac{1}{2+} \frac{1}{6+} \frac{1}{1+} \frac{1}{1+} \frac{1}{7+} \frac{1}{3+} \frac{1}{2}$. The successive convergents are $\frac{2}{1}, \frac{1}{3}, \frac{3}{8}, \frac{19}{7}, \frac{122}{55}, \frac{141}{52}$. Let us see what these convergents signify. (a) The convergent $\frac{19}{7}$ means that on an average there are 7 Adhikamāsās per 19 years. This ratio was adopted in the Romaka Siddhānta of Panchasiddhāntika. It means that $19 \times 12 = 228$ Solar months are equal to 235 lunations. The Metonic cycle described in modern astronomy is based upon this equivalence. The recurrence of Moon's phases in 19 years ie correspondence of the Moon's phases or tithis with the dates of the English year and Meton's formula are based on this equivalence. Recurrence of phase means recurrence of the relative positions of the Sun and the Moon, which again means recurrence of the S'uddhi, for S'uddhi is no other than the interval between the New Moon and Samkrānti. In the next verse Bhāskara says that a Kshayamāsa recurs after a lapse of either 19 years or 122 years or 141 years. The numerators of the last three convergents are 19, 122 and 141 which are the numbers of Solar years that effect recurrence of the same S'uddhi and as is going to be mentioned shortly a S'uddhi of 21 tithis is likely to bring in a Kshayamāsa. Hence a Kshayamāsa recurs either in 19 years or 121 years or 141 years.

2. Mention of the occurence of a Kshayamāsa was made by S'ripati first and not by the preceding astronomers. However, mention of it is there in the

literature and it is not clear whether observance of this Kshayamāsa was defunct for some centuries in between.

3. We shall now proceed to see how there occur two Adhikamāsas on either side of a Kshayamāsa. A simple argument is as follows. The convergents cited above namely $\frac{19}{7}$ or $\frac{122}{45}$ or $\frac{141}{52}$ signify that either 7 or 45 or 52 Adhikamāsās are to occur in the course of 19 or 122 or 141 Solar years normally. But when the S'uddhi happens to be 21 days at the beginning of the Solar year, it so happens that the S'uddhi goes on increasing for the first five months because the Sun is in his apogee when his longitude is 78°, and his motion being slow for three months when he is on either side of his apogee, the Moon gains over him in shorter intervals of time and as a consequence the lunations are of shorter duration. This means that the S'uddhi goes on increasing during those months and rapidly increases from its value 21 at the beginning to 30 by about Bhādrapada month. Under these circumstances, a Samkramaṇa occurs generally just before the beginning of Bhādrapada. The next Samkramaṇa happens just a little after the lapse of Bhādrapada, so that the month of Bhādrapada goes without a Samkramaṇa and as a consequence, it becomes an Adhikamāsa. Thus far it is alright that an Adhikamāsa has occured as per the meaning of the convergents. But when the Bhādrapada thus becomes an Adhikamāsa, the subsequent months from Kārtica to Mārgasira, being of longer duration than the corresponding Solar months, the Sun having a quicker motion on either side of his perigee, there is every likelihood of a Solar month being contained between two conjunctions or New Moon days. In other words two Samkramaṇas occur either in Kārtica or Mārgasira or Pausha. This means that as per the convention for the occurence of a Khayamāsa, one of the aforesaid lunations must become a Kshayamāsa. Thus the Adhikamāsa which

is due to occur during the course of the year, though it has occured has been lost. So, to make amends, another Adhikamāsa is to occur as is warranted by the convergents cited above. It might be asked what if two Adhikamāsās occur and why a Kshayamāsa be instituted at all. The reason is not that a religious convention warrants it but because the wedding of the luni-Solar year to the Solar year has to be made on a particular principle. Normally, so long as a Samkramaṇa goes on occuring during the course of a lunation, the two systems of reckoning may be seen to be running parallel. But if a particular lunar month does not contain a Samkramaṇa it is to be taken as a warning that the luni-Solar reckoning has overtaken the Solar by one lunation. This lunar month has to be curtailed to make the two kinds of reckoning to proceed side by side. This convention naturally raised the qnestion as to how to deal with a lunation which contains two Samkrāntis. The Solar month their has to be deleted to make the two systems run concurrently. This deletion of a Solar month is achieved not by declaring the particular Solar month as an Adhikamāsa but what is virtually the same two lunar months are deemed to lapse during the course of that solar month. This kind convention helped the occurrence of one Adhikamāsa alone as scheduled because one of the two Adhikamāsās has been nullified by the convention of a Kshayamāsa.

Why the proposition that a Kshayamāsa generally occurs when the S'uddhi at the beginning of the Solar year happens to be 21 tithis is quite evident because in such a case generally Bhādrapada becomes an Adhikamāsa, which again entails the occurence of two Samkramaṇas during the course of one of the three lunations beginning with Kārtica, which happen to be longer than the corresponding Solar months. In other words a Kshayamāsa is [illegible] to occur only when Bhādrapada happens to be an [illegible]māsa, and this in turns happens only when [illegible] to be 21 at the beginning of the Solar year. Adhikamāsās do occur when the S'uddhi happens to be more than 21, but in this case one of the months prior to Bhādrapada happens to be an Adhikamāsa and then by the time Kārtica is reached, the Samkramaṇa occurs not at the beginning of Kārtica but a little later, which means that a second Samkramaṇa could not occur before the lapse of that lunation. This therefore will not be a Kshaya month.

The S'uddhi which is defined as the number of tithis in between a New-Moon and a Samkramaṇa, is clearly the interval which has been gained by the luni-Solar system over the Solar. This S'uddhi increases at the average rate of 11 days, 3 ghatis, 52 palas and 30 Vipalas per an year as we have seen already. For the occurrence of an Adhikamāsa during the course of an year the S'uddhi at the beginning of the year must be such that it accrues to a lunation during the course of the year. The apogee of the Sun being almost stationary having a very slow motion the lengths of the Solar months do not vary appreciably for a good number of years. Taking that the Sun's apogee is roughly at 80° longitude (78° according to Hindu Astronomy) the Sun's motion is less than his average from the moment when he has 350° longitude upto the moment when he has 170°, so that the Moon gains rapidly over him during this period. In other words the lunations during this period are of shorter duration i.e. the luni-Solar months of chaitra upto Sravana will be shorter in length so that S'uddhi increases rapidly from March upto August. Thereafter it will attain a stationary value just for a little time and then decreases for the next six months upto February. The increase, however, far exceeds the decrease and the balance of increase per year is as mentioned above is 11–3–52–30.

Bhāskara mentions in the course of the commentary that (1) the average length of a lunation is 29 days, 31 ghatis and 50 palas; (2) the average length of a Solar month is 30–26–17 (3) when the daily motion of the Sun happens to be 61′, then the Solar month will have a length of 29-30 and as such falls short of a lunation and (4) that the minimum length of a Solar month is 29-20-40 only. Then he gives a general and broad explanation as to how a Kshayamāsa occurs on the following lines. Let us call NS as the interval between a New Moon and the subsequent Samkrānti. In other words it is the S'uddhi. Suppose Bhādrapada goes without a Samkrānti thus becoming an Adhikamāsa. Then in Āswayuja NS will have a small value, because the Samkrānti S which should have occured at the general rate of one per a lunation, has not occured before the lapse of Bhādrapada and being belated a little must have occured close on the heels of the New Moon of Bhādrapada. Let this interval NS have a particular value say x. Thereafter the Solar months grow gradually smaller i.e. the Samkrāntis occur earlier; so in Kārtica the value of NS decreases. It might decrease to such an extent that the next Samkrānti might occur before the next New Moon. Suppose this does not happen in Kārtica; then in Margasira the value of this NS is still smaller and there is a greater likelihood of another Samkrānti occuring during this Mārgāsira. Then the value of NS being the smallest in the month of Pausha, another Samkrānti is bound to take place during this month at the latest. In other words S now occurs before the next N i.e. the Samkrānti occurs just a little before the next New Moon. Let now this SN be a small quantity say y; or what is the same the next NS or the S'uddhi will be nearly a lunation. There after the Solar months begin to gain in length i.e. S occurs later and later. This means NS gains in length. Being nearly of a length equal to a lunation, and now gradually increasing it is bound to exceed a lunation shortly thereafter. This again means that S occurs later than the next New Moon, which is to say that one more lunation goes without a Samkrānti. Thus it is that another Adhikamāsa occurs shortly after the occurence of a Kshayamāsa. Hence in the course of one year there occur two Adhikamāsās and Kshayamāsa which means again that the balance of an

Adhikamāsa is there in that year. In other words, the occurence of a Kshayamāsa during the course of an year as per a stipulated convention does not preclude the average occurence of an Adhikamāsa which is normally due after an average lapse of $32\frac{1}{2}$ Solar months. That the occurence of a Kshayamāsa entails the occurrence of two Adhikamāsās on either side may be also seen in another way. Twelve lunations put together have a length of $354\frac{1}{3}$ days approximately during the course of which only eleven Samkrāntis could occur in case the initial S'uddhi is 21 days, for, subtracting this 21 from $354\frac{1}{3}$, there remain $333\frac{1}{2}$ days only which could contain only ten Solar months and not eleven. Ten Solar months are contained if there be only eleven Samkrāntis. Out of these eleven, two are consumed in a single lunation which happens to be a Kshayamāsa. Hence the remaining eleven lunations have to contain only nine Samkrāntis which means that two lunations have to go without Samkrāntis. In other words there are to be two Adhikamāsās during that year. Further both these Adhikamāsās could not occur on one side of the Kshayamāsa for the following reason. Since a Kshayamāsa could occur only during the course of Kārtica or Māgasirsha or Pansha which alone are longer than the corresponding solar months, in the event of both the Adhikamāsās taking their place on one side of these months, they are to take place in nine lunations which are on one side of the three months beginning with Kārtica. This means that only seven Samkrāntis are to occur during the course of $9 \times 29\frac{1}{2}$ days $= 265\frac{1}{2}$ days. Even seven Solar months fall short of this period so that it is impossible that only seven Samkrāntis should occur in this period. Hence, the two Adhikamāsās which are to occur during the year containing a Kshayamāsa have to take their place on either side of the Kshayamāsa

Bhāskara mentions that when the daily motion of the Sun equals 61′, the length of the Solar month will be $\frac{1800}{61}$ $= 29\text{–}30$ approximately. He also states that the minimum length of a Solar month will be 29–20–40. This will be so when the average daily motion during the course of a month is 61–20–30.

Verse 7. A mention of the years past and future that had and will have Kshayamāsās.

A Kshayamāsa occured in the Saka year 974 and will occur in the years 1115, 1256, 1378. Thus generally it occurs once in 141 years or even 19 years.

Comm. We have seen before that according to the convergents given above a Kshayamāsa generally occurs in 19 years or 122 years or 141 years. In the course of the commentary Bhāskara says "19 years before or after" so that 141 − 19 = 122 years was also meant by him. He gives how in the course of 141 years or 19 years the S'uddhi at the beginning of the year happens to be approximately 21 days. In this behalf, he invokes his formulation of the Adhikamāsās in the verse 6 under Pratyabdas'uddhi. As per that formula the number of Adhikamāsās in 19 years will be 7–13–37–30 taken by Bhāskara as 7–13–40. In other words the S'uddhi increases by 13 ghatis and 38 palas from what it was 19 years ago; thus practically the value of the S'uddhi recurs in periods of 19 years which means that if a Kshayamāsa occurred this year, there is a likelihood of its recurrence after nineteen years. Of course the recurrence would not be certain because there is an excess of 13 ghatis S'uddhi which might preclude its occurrence. Similarly in 122 years and 141 years the numbers of Adhikamāsās would be respectively 45 minus 7–15 ghatis and 52 plus 6 ghatis. 22 palas and 30 Vipalas, the latter being taken by Bhaskara to be 52 Adhikamāsās plus 6 ghatis and 20 palas. Thus Kshayamāsas are more and still more likely to recur in intervals of 122 years and 141 years respectively, the S'uddhis at the beginning of the year almost recurring and assuming the original value of 21 days for a Kshayamāsa to occur.

Bhāskara adds that a Kshayamāsa occured in 974 S'aka i.e. 62 years before his birth which knowledge he must have derived from hearsay or even by calculation. He then predicted its recurrence in the Saka years 1115, 1256 and 1378 which were at intervals of 141 years, 141 years and 122 years respectively. In the year 974 Saka i.e. 4153 Kali year, applying the above formula Dwidhābdāḥ the number of Adhikamāsās elapsed from the beginning of the Kali were 1531–21–12–52–30 i.e. the S'uddhi at the beginning of that year was 21–12–52–30. This naturally entailed the occurence of a Kshayamāsa. Thereafter during the years 1115, 1256 and 1378, the S'uddhis in the beginning of the years should be as per the above analysis 21–12–52–30 plus 68–22–30 i.e. 21–19–15; 21–25–37–30 and 21–25–37–30 and 21–18–23 respectively so that Bhāskara could forecast the occurrence of the Kshayamāsās in those years also. In this context it is worth-noting that Gaṇes'a notified in a verse that Kshayamāsās would recur in the years 1462, 1603, 1744, 1885, 2167, 2232, 2373, 2392, 2524, 2533, 2655, 2674, 2796, 2815 according to Sūrya Siddhānta and according to Aryabhṭīya during the years 1482, 1793, 1904, 2129, 2186, 2251. These years also may be verified by computing the S'uddhis as said before and also with respect to Aryabhaṭīya according to which the number of Adhimāsas during a yuga differs by a little and as such effects a difference in the sequence of the Kshayamāsa years.

Verse 8. Tell me, how, when and in the course of how many years do two Adhikamāsas occur as mentioned by the Rishis? Questioned accordingly by an expert in questioning, if a mathematician could know the answer, I would reckon him as no other than Bhāskara (either the Sun-god or he himself i.e. Bhāskarācharya) who could make the lotus-buds of mathematicians blossom.

Comm. Probably taking up this challenge alone Gaṇes'a answered the question and gave the years cited above which should bring in a recurrence of Kshayamāsās.

N.B. If does not suffice merely to compute the S'uddhis alone in the beginning of the years but a rigorous computation necessitates the calculation of the moments of New Moons and Samkrāntis also during the particular years as well, in as much as, the motion of the Moon also comes into the picture and it differs from month to month on account of the rapid motion of his apogee.

Here ends the section named Adhimāsādi-nirṇaya.

THE SECTION BHŪ-PARIDHI-MĀNĀDIKA-
CIRCUMFERENCE OF THE EARTH

Verse 1. The circumference of the earth's globe is 4967 yojanas; its diameter 1581. A yojana is equal to $\frac{d \times 360}{\mathrm{?}}$ where $\delta\phi$ is the difference in the latitudes of two places on the same terrestrial meridian in degrees, C the circumference of the earth's globe given above and *d* the distance between the two places.

Comm. The second half of the verse gives the method of computing the circumference of the earth's globe, which is mathematically correct; for, by the rule of three "If by a difference of 90° in latitude we have $\frac{1}{4}$ C, what shall we have for 1° difference in latitude?" The answer is $\frac{C}{360}$. Again "If by distance of d between two places on the same meridian, we have a difference of $\delta\phi°$ in latitude, what shall we have for 1° difference in latitude?' The answer is $\frac{d}{\delta\phi}$. Both the answers must be the same; so equating them, we have $\frac{C}{360} = \frac{d}{\delta\phi}$ = number of yojanas = x say. Hence one yojana $= \frac{x}{x} = \frac{d}{\delta\phi} \div \frac{C}{360} = \frac{d \times 360}{C \times \delta\phi}$

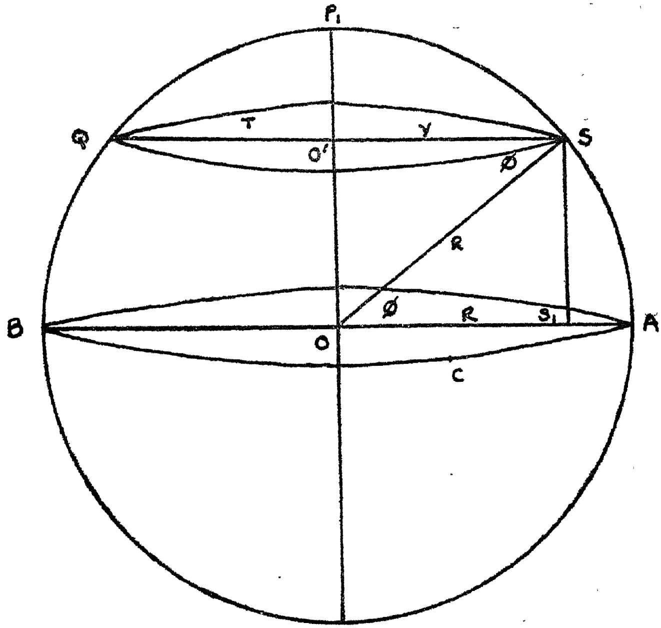
N.B. Here it must be noted that a yojana's length is derived from the number of yojanas contained in the circumference as reported in the Āgama.

In the course of the commentary under this verse, Bhāskara explains why he had recourse to this kind of definition, which is based upon āgama and as such does not contain a proof. He says that in as much as the definition of a yojana was given basing ultimately on the units of Angulas and yavas (grains of paddy) differently by different authorities, and in as much as the circumference of the earth's globe was more or less unanimously accepted, he has chosen to define a yojana on the accepted measure of the circumference. In some other place Bhaskara says that a yojana is equal to 4 Krōsās, where the word Krōsa etymologically means that distance through which the topmost voice of a healthy person could be heard by another healthy person having good audition.

In this context it may be recalled how S'ripati defined a yojana under verses 69, 70 Madhyamādhyāya of his Siddhānta S'ekhara. " The minute speck of dust that is seen flying in the light of the Sun's rays entering a house through the windows, is called a paramāṇu (not an atom as is being translated now). Eight such paramāṇus equal one Reṇu. Eight Reṇus equal the breadth of the end of a hair known as Vālāgra or Kaccha-mukha or yūka. Eight Vālāgras equal one Likshā; Eight Likshās equal one Yava; eight Yavas equal one Angulā; twelve Angulās make what is called a Vitasti (i.e. the expanded length of a palm of the tallest person). Two Vitastis equal one Hāsta. Four hastās make one Chāpa. Two thousand Chāpas make one Krōs'a and four Krōs'ās are reported to be equal to a yojana by Astronomers "

Verse 2. Rectification of the circumference .ie. finding the length of the circumference of the earth parallel to the terrestrial equator and passing through the locality in question. Also the definition of the primary meridian.

The equatorial circumference of the earth multiplied by cos ϕ and divided by R or multiplied by 12 and divided by the hypotenuse of the right angled triangle formed by the gnomon and the equinoctial midday shadow thereof. (Hereafter called equinoctial hypotenuse) gives the circumference of the earth parallel to the equator and passing through the locality (hereafter called the rectified ference). Also the primary terrestrial meridian is that longitudinal line passing through the places (1) Lanka (2) Ujjain (3) Kurukshetra and (4) the north pole.



Comm. (Ref. fig. 4) Let ABC be the terrestrial Equator, and QST be a circle parallel to this terrestrial equator and passing through the locality S. This QST is called the rectified circumference of the earth at the place S. The terrestrial equator is defined as follows. It is known that the earth rotates about herself about the axis $P_1 P_2$ where $P_1 P_2$ is called the polar axis or Dhruvayashti. In other words the entire heavens appear to revolve round the earth in such a way that any star will appear to be revolving in a circle called its diurnal circle which is parallel to circle ABC. The points P_1 and P_2 are called the north south pole respectively. These two points evidently do not move, though the earth is herself rotating in the clockwise direction. If O be the centre of the earth, OP_1 produced meets the skies at a point called the north celestial pole and OP_2 produced meets the skies in the point called the south pole. It so happens that the north celestial pole is very near a star which is called the pole star. This star is known as the Dhruva-Tāra in as much as it does not appear to move at all while all the heavens (i.e. all the stars of the sky) appear to be revolving round the earth rising and setting as seen at any place. (The word Dhruva means fixed). Aryabhaṭāchārya mentioned in so many words that it is the earth that really rotates and so the stars which are themselves fixed appear to be going round the earth in circles parallel to the circle ABC. अनलो

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॥ (Explained under verse 7 Bhagaṇādhyāya). The celestial Equator is the great circle which is the circle of intersection of a plane perpendicular to the polar axis and passing through the earth's centre with the celestial sphere (celestial sphere is the sphere-like surface which shape the Sky takes and on which the stars and the planets appear to be studded.) Similarly the terrestrial equator ABC is the circle of intersection of the earth's globe with the same plane. Thus the terrestrial and the celestial equators are concentric coplanar circles with the earth's centre as the common centre. A great circle of a sphere is a circle whose plane passes through its centre. Thus ABC is a great circle on the earth's surface, because its plane passes through the earth's centre. Similarly the celestial equator is a great circle of the skies. The circle QST is called a small circle, just as the diurnal circle traced by any star in its diurnal rotation is also a small circle parallel to the celestial equator. Thus small circles, an infinity of them can be drawn parallel to the terrestrial equator ABC and they will be in decreasing dimension as we proceed towards the pole. Hence the Sphutaparidhi or the rectified circumference

QST at the locality S is a small circle whose circumference is smaller than that of ABC. The problem is now to find the length of this circle QST. Evidently $\frac{QST}{ABC} = \frac{2\pi r}{2\pi R}$

$= \frac{r}{D}$ where r and R are respectively the radii of the two

circles. But $\frac{r}{R}$ = Cosϕ from the triangle OSO′, where

$O'_1SO = SOA$ = latitude of the place S. QST =
ABC × Cosϕ = $\frac{ABC \times H \cos\phi}{R}$ I

the Hindu cosine of ϕ known as lambajyā and stands for $OS_1 = O'_1S = r$. This lambajyā is also called Dyujya if the small circle is the diurnal circle of a Star. The diurnal circle of a Star is called Dyujya-Vritta and its radius is called Dyujyā-Equation I proves the first statement of the

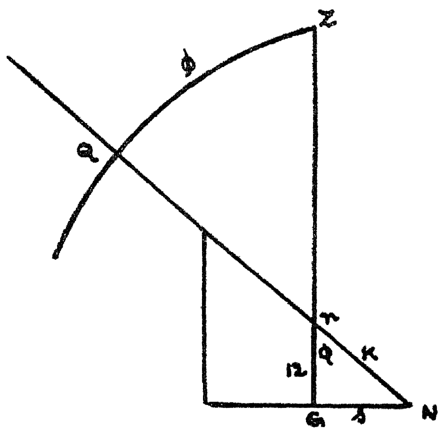

Fig. 5

verse. In fig. 5 Gn N is called the fundamental gnomonic triangle where Gn is the vertical gnomon pointing to the Zenith Z of the celestial sphere and is considered to be of 12 units (Angulas as they are called), GN the midday-shadow of the gnomon cast on an equinoctial day when the

Sun is at the point Q where Q is the point of intersection of the celestial equator with the meridian of the place. If the equinoctial shadow be denoted by 's' and the hypotenuse of the triangle nGN namely nN be denoted by K (called the Vishuvatkarṇa or the equinoctial hypotenuse) then $\frac{s}{K} = \text{Sin}\,\phi$ and $\frac{12}{K} = \text{Cos}\,\phi$. If H Sin ϕ be the Hindu sine of ϕ called the Akshajyā H Sin ϕ = R Sin ϕ

so that $\frac{s}{K} = \frac{\text{H Sin}\,\phi}{R}$ and $\frac{12}{K} = \frac{\text{H Cos}\,\phi}{R} = \frac{\text{Lambajyā}}{\text{Trijyā}}$.

Substituting for $\frac{\text{H Cos}\,\phi}{R}$ the value $\frac{12}{K}$ in equation I above, we have

$$\text{QST} = \text{Sphuta - paridhi} = \frac{\text{Bhū - paridhi} \times 12}{K} \qquad \text{II}$$

which proves the second statement given in the verse.

Regarding the third statement, which defines the Bhu-Madhya-Rekhā (In modern text books of geography the terrestrial equator is spoken of as the Bhu-Madhya-Rekha. The terrestrial equator is called Niraksha-Rekha in Hindu Astronomy which means the circle of Zero-latitude), it is the primary meridian taken by Hindu Astronomers. In modern astronomy the primary meriodian is taken as the Greenwich meridian. Bhāskara has given four places located on the Hindu primary meridian, but, S'ripati gives many more places located on this primary meridian under verse 96 Madhyamādhyāya namely (1) Lankā (2) Kanyākumāri (3) Kānchī (4) Pannāta (5) the six-faced white mountain (6) Sri-Valma-gulmam (7) Māhishmatī (8) Ujjain (9) An Āsrama (10) Pattasiva, a town (11) Sri Gargarāna (12) Sthānvīswara known also as purohita (13) S'ītagiri and (14) Sumeru. Some of these places cannot be properly identified but the following remarks may be made (a) Pannāta is one of the fifty-six small countries into which India was divided in ancient times according to the purānic literature

(b) It is not clear what places are indicated by (5) and (6) cited above (c) In some works Māhishmatī and Ujjain are used synonymously. (8) is not clear. Regarding (9) there is one pattas'iva near Rajahmundry but S'ri pati does not seem to have meant it. Again (10) is not clear, Regarding (11) it is to be noted that the place is now pronounced (probably mis-pronounced) as Sthānes'wara. If (12) means the Himalaya mountain, there is not much meaning to say that it lies on the primary meridian; only a cross-section of it could lie there upon. The entire mountain extends from west to East over more than a thousand miles.

Verse 3. To find the correction known as Des'āntara.

The distance between two places on the same latitude multiplied by the daily motion of a planet and divided by the rectified circumference is a correction subtractive in the east and additive in the west of the primary meridian in the planetary position obtained.

Comm. In Hindu Astronomy the mean planetary positions are first calculated for the Sun-rise at the primary meridian. Now suppose a place lies to the east of this meridian. Then the Sun-rise at the place happens to occur earlier than on the primary meridian. Hence the correction in the mean computed position of the planet is negative if the position were to be calculated for the local Sun-rise. If the place happens to be on the western side of the primary meridian the reverse holds good i.e. the correction is to be additive. The amount of the correction is the amount of the motion of the planet in between the two Sun-rises. Let the planet move an arc equal to δm per day i.e. it moves δm when the earth rotates once about her axis. The time between the two Sun-rises above is the time by which the local meridian is carried through the distance between the locality and the primary meridian's point of inter-section with the latitudinal line or what is the same through the arc of the rectified circumference of the earth pertaining to the locality. If d be this distance then the rule of three to be used is "If the length of the rectified circumference viz. C rotates by the time the planet moves a distance δm, what is the arc traversed through by the planet if an arc 'd' of C rotates through?" The answer is $\frac{d \times \delta u}{C}$ which is the correction required.

Verses 4, 5, 6. The Correction Des'āntara expressed in time.

The eclipse of the Moon occurs at a place situated on the east of the primary meridian later than on the primary meridian and vice versa. The time in between the two moments is the Des'āntara expressed in time. The distance of Des'āntara ie. the distance of the locality from the primary meridian measured along a parallel to the terrestrial equator or Niraksha Rekhā is obtained by multiplying the rectified circumference by the Des'āntara measured as above in ghatis and dividing by 60. Also the above time in ghatis multiplied by the planets' daily motion and divided by 60, gives the correction in arc in the computed mean planetary motion.

Further the week-day begins after or before the local Sun-rise by that Des'āntara expressed in time according as the locality is on the east or west of the primary meridian. Also the week-day begins after or before the local Sunrise by the ghatis of the correction known as chara according as the Sun is in the northern or Southern hemisphere.

Comm. An eclipse is first computed for the primary meridian. If an observer wants to know whether he lies east or west of the primary meridian and to know the Des'āntara correction in time, the following procedure is to be adopted. Let a lunar eclipse begin x ghatis after the Sun-rise of the primary meridian. Let the observer note the time y ghatis which have elapsed after Sun-rise at his own place when the eclipse begins. Since a lunar